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Structural Behaviour of Closely Packed Inverted Egg-Shaped Sewer Linings During Installation and Under Various Restraint Conditions

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ABSTRACT

The paper describes the results of a numerical parametric study aimed at studying the structural response of closely packed inverted egg-shaped sewer linings. The effect of various restraint conditions which simulate different temporary support systems that may be used by the contractors during the installation of the lining, and of different loading configurations which may arise at different stages of grouting the annulus gap between the lining and the sewer, have been thoroughly investigated. Covering the feasible range of geometric, material and loading parameters, comprehensive design curves, based on the allowable stress- and deflection-limit criteria, are presented. A comparison between the various types of restraints leads to enhancement factors for the permissible grouting pressure or, alternatively, to reduction factors in terms of the lining thickness that could be used in designing lining systems. Finally, the structural response of inverted egg-shaped sewer linings is compared with their egg-shaped counterparts. © 1997 Elsevier Science Ltd.

NOTATION

A	Dimensionless constants for maximum bending stress (staged grouting)
B_x, B_y	Dimensionless constants for maximum deflection (staged grouting)

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C	Dimensionless constants for maximum bending stress (flotation)
D_x, D_y	Dimensionless constants for maximum deflection (flotation)
E	Dimensionless constants for maximum bending stress (uniform pressure)
E_s	Short-term modulus of elasticity of lining material
EF	Enhancement factor for allowable grouting pressure
FE	Finite element
F_{cr}	Critical axial force in circular lining
F_x, F_y	Dimensionless constants for maximum deflection (uniform pressure)
G	Unit weight of grout mix
GRC	Glass-reinforced cement
GRP	Glass-reinforced plastic
H	Excess head of grout (measured form crown of lining) corresponding to uniform pressure load
h	Height of lining
K	$\left(\frac{Gw}{E_s}\right)\left(\frac{w}{t}\right)^3$
M_x	$D_x + F_x(H/w)$
M_y	$D_y + F_y(H/w)$
N_x	$D_x + F_x\left(\frac{p}{Gw} - \frac{h}{w}\right)$
N_y	$D_y + F_y\left(\frac{p}{Gw} - \frac{h}{w}\right)$
p	Allowable grouting pressure (measured from invert of the lining)
q	Uniform pressure intensity
q_{cr}	Critical uniform pressure
R	$(S_S/Gw)(t/w)^2$
RF	Reduction factor for minimum permissible lining thickness
S-L	Stress-limit criterion
D-L	Deflection-limit criterion
S_s	Allowable short-term bending stress of lining material
S_t	Total bending stress of lining material due to combined flotation and external pressure
t	Thickness of lining
WRC	Water Research Centre
w	Width of lining
δ	Deflection of lining
δ_t	Total deflection of lining material due to combined flotation and external pressure

1 INTRODUCTION

Strictly speaking, one should not refer to linings as a new concept. What is new is the rapid growth in their use as a means of controlling seepage from hydraulic facilities. Also new is a developing awareness by users and designers that there is a separate and important technology concerned with the use of linings. When the subject of linings is mentioned, the common reaction is to think of the reservoir, canals, concrete and steel tanks. What is uncommon, is to think of the linings of sewers. The huge capital expenditure and indescribable sufferings of traffic during the replacement of old existing sewers has drawn the engineers out of the traditional methods and led them to seek a better and easier solution, i.e. the lining of existing sewers. In the past, the structural behaviour of egg-shaped sewer linings under installation and operational conditions have been studied.[1–3] The present study concentrates on the behaviour of inverted egg-shaped sewer linings.

2 LINING TECHNOLOGY

The linings are usually made of glass-reinforced plastic (GRP) or glass-reinforced cement (GRC). Steel linings are also used. Obviously, an inverted egg-shaped lining (see Fig. 1) is to be inserted into the similarly shaped sewer after allowing for an annulus gap so that the sewer lining fits

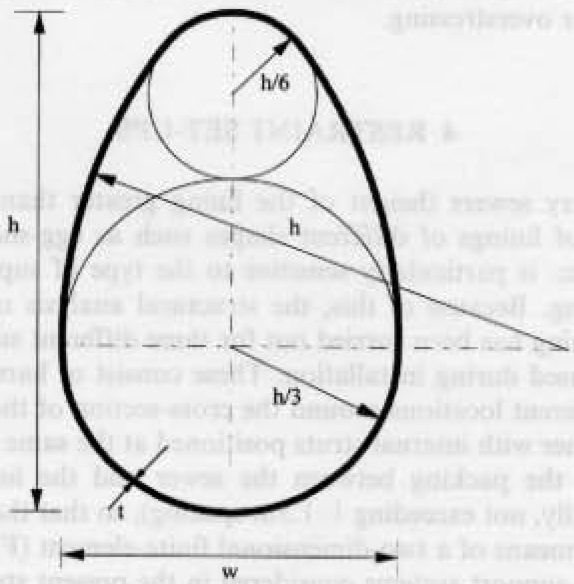

Fig. 1. Inverted egg-shaped lining: the shape of the lining used in the analysis.

within the existing sewer with a roughly uniform gap between the lining and the sewer walls. The gap between the lining and the sewer is then filled with a cementitious grout which, when set, creates a composite sewer-lining structure.

3 TECHNIQUES OF GROUTING

In sewer lining, there are two techniques of grouting which are generally adopted. They are described in the following.

3.1 Staged or partial grouting

In this method of grouting, grouting is performed in two stages. The first stage involves grouting the annulus up to the springings, and this is followed by a second stage carried out after the grout of the first stage has set.

3.2 Full grouting

In this technique, full grouting is performed in a single stage. This technique is more practical than staged grouting. However, during full grouting, the lining is subjected to higher pressure so that a thicker lining or additional supports may be deemed essential in an effort to avoid excessive deformation or overstressing.

4 RESTRAINT SET-UPS

With man-entry sewers (height of the lining greater than 900mm), the performance of linings of different shapes such as egg-shaped, inverted egg-shaped, etc. is particularly sensitive to the type of support provided during grouting. Because of this, the structural analysis of the inverted egg-shaped lining has been carried out for three different support systems that may be used during installation. These consist of hardwood wedges packed at different locations around the cross-section of the lining on the outside, together with internal struts positioned at the same locations. It is assumed that the packing between the sewer and the lining is closely spaced (typically, not exceeding 1–1.5m spacing), so that the structure can be studied by means of a two-dimensional finite-element (FE) model. The three possible support systems considered in the present study are shown in Fig. 2.

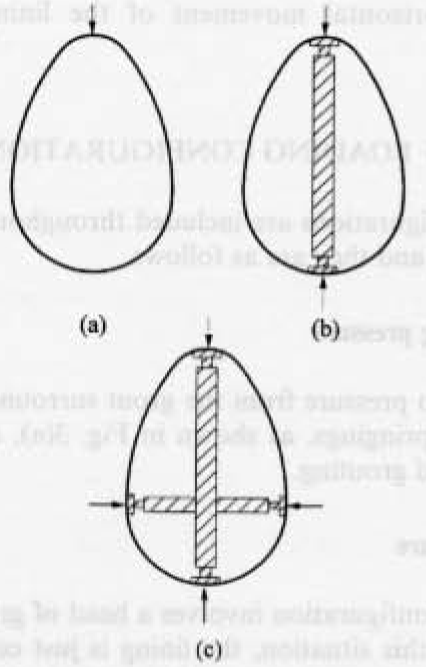

Fig. 2. Inverted egg-shaped lining: the support systems studied. (a) Boundary case 1; (b) boundary case 2; and (c) boundary case 3.

4.1 Boundary condition 1

It consists solely of a restraint at the crown (top) of the lining as shown in Fig. 2(a). It is to be noted here that, normally, grout is injected through the invert (bottom) of the lining. As grout moves forward and upward during the injection of grout, this may cause the lining to go upward and thereby reduce the annulus gap between the sewer and the lining. This is why a restraint at the crown is always expected.

4.2 Boundary condition 2

The second support system, as shown in Fig. 2(b), comprises restraints at both the crown and the invert of the lining. Like the boundary condition 1, boundary condition 2 imposes restraints on the vertical movement of sewers, not only of the crown, but also of the invert. This boundary condition is vertically stiffer than the former.

4.3 Boundary condition 3

This form of support consists of restraints at the crown, invert and springings of the linings (Fig. 2(c)). In addition to vertical restraints, it restricts the horizontal movement of the lining at some specific points.

5 LOADING CONFIGURATIONS

Three loading configurations are included throughout the analysis unless otherwise specified and they are as follows.

5.1 Staged-grouting pressure

This corresponds to pressure from the grout surrounding the lining up to the height of the springings, as shown in Fig. 3(a), and so simulates the first phase of staged grouting.

5.2 Flotation pressure

This type of load configuration involves a head of grout up to the crown, as in Fig. 3(b). In this situation, the lining is just covered by grout and, hence, the buoyancy force acting on the lining is the maximum that can

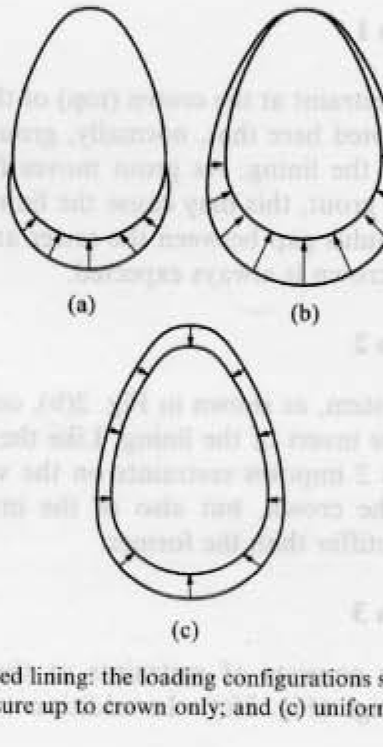

Fig. 3. Inverted egg-shaped lining: the loading configurations studied. (a) Staged grouting; (b) pressure up to crown only; and (c) uniform pressure.

occur. For this reason the loading corresponding to Fig. 3(b) is sometimes called the flotation pressure.

5.3 Uniform pressure

The third load configuration, shown in Fig. 3(c), corresponds to the uniform pressure which is applied on the lining as a consequence of an excess head of grout. Flotation pressure and uniform pressure can be superimposed in order to simulate any grout pressure applied on the lining during full grouting.

6 CALCULATION OF LOADS

For each load configuration and boundary case, the parametric study is carried out by varying one parameter at a time, whilst keeping the others constant. The results are most conveniently given in terms of dimensionless equations linking all the independent parameters together. Such equations are derived on the basis of a curve-fitting exercise. In considering this, it should be noted that, in the case of the loading corresponding to flotation and the first phase of staged grouting, the applied load is defined by the lining height, h, and the specific weight of the grout mix, G. In these two loading cases, the applied pressure at any point on the lining can be calculated by multiplying the specific weight of the grout mix by the distance from the top of the grouting to the point at which the pressure is calculated. For the uniform-load case, on the other hand, the external load is defined by the values of excess head of grout, H (and its specific weight G) and is independent of the height of the lining.

7 BASIS OF DESIGN

During installation the lining is subjected to grout pressure. In some cases, this may lead to an overstressing of the lining at different sections, which may cause total collapse of the linings. Alternatively, excessive deformation of any part of the lining might occur, affecting the serviceability of the relined sewer. Therefore, a properly designed sewer must satisfy both stress- and deflection-limit criteria. Here, the stress-limit criteria is so defined that the maximum bending stress developed during grouting must not exceed the allowable bending stress of the lining material. For deflection-limit criteria, a maximum allowable deflection in the lining not exceeding 3% of the width of the lining (as advocated by the Water Research Centre in its *Sewerage Rehabilitation Manual*[4]) has been followed.

8 PARAMETERS TO BE USED IN THE ANALYSIS

The parameters included in the subsequent analysis are divided into geometrical, material and load parameters. These are as follows:

(1) Geometrical parameters:
w = width of lining
h = height of lining
t = thickness of lining

(2) Material parameters:
S_s = allowable short-term bending stress of lining material
E_s = short-term modulus of elasticity of lining material

(3) Load parameters:
G = unit weight of grout mix
H = excess head of grout measured from crown of lining corresponding to uniform-pressure load.

9 MATHEMATICAL FORMULATION OF THE ANALYSIS

As mentioned earlier, it is advantageous and convenient to express the results of the analysis in terms of non-dimensional equations encompassing all the parameters involved in the analysis. Hence, the design curves that will be proposed after an extensive parametric analysis of the inverted egg-shaped sewer linings can be used for all types of lining materials and lining sizes of that specific shape. The dimensionless equations corresponding to the bending stress S and the deflection δ at any point on the lining can be written for the three load cases as follows:

(1) Staged grouting (Fig. 3(a))

$$S/Gw = A(w/t)^2, \tag{1}$$

$$\delta/w = (B_x^2 + B_y^2)^{1/2} K. \tag{2}$$

(2) Flotation (or pressure up to the level of crown) (Fig. 3(b))

$$S/Gw = C(w/t)^2, \tag{3}$$

$$\delta/w = (D_x^2 + D_y^2)^{1/2} K. \tag{4}$$

(3) Uniform pressure (excess head H) (Fig. 3(c))

$$S/Gw = E(H/w)(w/t)^2, \tag{5}$$

$$\delta/w = (F_x^2 + F_y^2)^{1/2}(H/w)K, \tag{6}$$

where

$$K = (Gw/E_s)(w/t)^3. \tag{7}$$

In these equations, S/Gw can be regarded as a non-dimensional stress while δ/w is the deflection related to the size of the lining and K is a measure of lining flexibility. Here, A, C, E, B_x, B_y, D_x, D_y, F_x and F_y are all constants which depend on the boundary set-up adopted during the grouting of the annulus and loading configurations used in the analysis.

The total bending stress S_t and the total deflection δ_t at any point in a lining subjected to a head of grout which is greater than the lining height, h (i.e. full flotation), can be divided into values of bending stress and deflection resulting from the two loading cases of pressure up to the crown (i.e. flotation) and uniform pressure. This implies that, by adding eqns (3) and (5), and eqns (4) and (6), the following dimensionless equations for the total bending stress and the total deflection can be written as

$$S_t/Gw = |(C + E(H/w))(w/t)^2| \tag{8}$$

$$\delta_t/w = (M_x^2 + M_y^2)^{1/2} K \tag{9}$$

where

$$M_x = D_x + F_x(H/w) \tag{10a}$$

$$M_y = D_y + F_y(H/w) \tag{10b}$$

As was previously mentioned, since the maximum bending stress and the maximum defection in a lining must not exceed the respective values of S_s and $0.03w$, the values of S_t and δ_t in eqns (8) and (9) can be replaced by S_s and $0.03w$, respectively. As the point of injection of the grout is usually located at the invert of the lining, it is convenient to replace the value of H in eqns (8) and (9) by the equivalent expression $(p/G)-h$, where p is measured from the invert of the lining. As a result, eqns (8) and (9) can be rewritten to produce the following design equations:

$$R = |C + E(p/Gw - h/w)| \tag{11}$$

where

$$R = (S_s/Gw)(t/w)^2, \tag{12}$$

and

$$0.03/K = N_x^2 + N_y^2)^{1/2}, \tag{13}$$

where

$$N_x = D_x + F_x(p/Gw - h/w) \tag{14a}$$

$$N_y = D_y + F_y(p/Gw - h/w). \tag{14b}$$

For any particular lining geometry and material properties, the above equations must be satisfied at the locations of maximum bending stress and deflection in the lining. This, in turn, determines the maximum allowable grouting pressure, p, which can be applied to the lining during grouting. [It should be pointed out that all the previous equations for the inverted sewer linings take exactly the same form as their counterparts for the (non-inverted) egg-shaped linings (which are listed in Ref. [1]), except that, in eqns (11) and (14), h/w has not been replaced by 1.5 (see Fig. 1); this facilitates a comparison between the two types of lining.]

10 TWO-DIMENSIONAL FINITE-ELEMENT MODEL

A linear two-dimensional FE model is used in order to simulate the behaviour of inverted egg-shaped linings under various probable loads during installation. The shape of the lining used in the analysis has already been shown in Fig. 1. The thickness of the lining is assumed to be constant all around the cross-section. Due to symmetry of the lining geometry, loading and boundary conditions about the vertical axis (i.e. Y-axis), only half of the cross-section, shown in Fig. 4, is analysed.

The elements used in the analysis are two-noded beam elements, each having three degrees of freedom (horizontal and vertical displacement, and rotation) at each node. The mesh adopted consists of 25 elements, the node numbers corresponding to the crown, springing and invert being 26, 11 and 1, respectively.

The restraints due to the support system, shown in Fig. 2, are simulated numerically in the analysis by fixing the horizontal and vertical components of displacement at the corresponding nodal points. This involves a small approximation in that the deformation in the restraining struts is ignored, the strut being very stiff compared with the lining. As half of the cross-section is analysed, the horizontal and rotational components of displacements at nodes 1 and 26 of the lining are restrained. In addition, in Fig. 4, the vertical displacement at node 26 is set to zero for boundary case 1 and the vertical displacements at nodes 1 and 26 are made equal to zero for boundary case 2. Similarly, restraints have been imposed on the vertical displacement at nodes 1 and 26, and on vertical and horizontal displacements at node 11 (springing) in order to simulate boundary case 3.

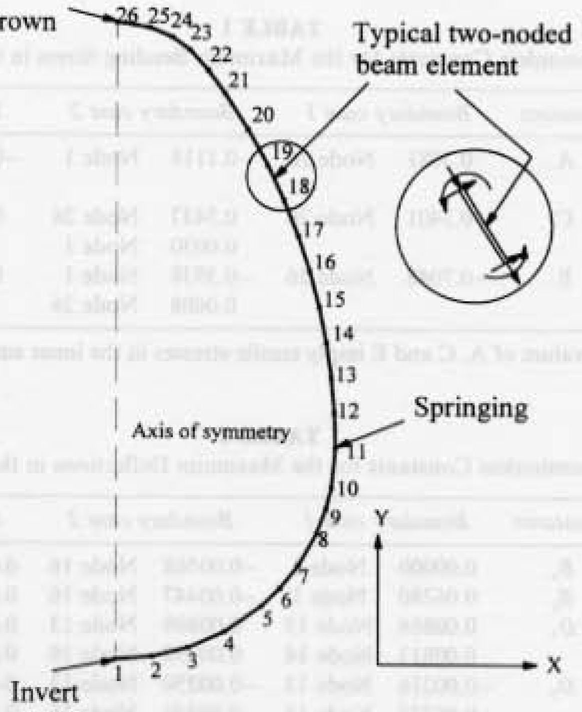

Fig. 4. Inverted egg-shaped lining: two-dimensional FE mesh adopted in the analysis.

The various loading configurations, shown in Fig. 3, have been simulated by applying equivalent point loads at the appropriate nodes.

11 COMPUTATION OF CONSTANTS

For each load and boundary case, the parametric analysis is carried out by varying one parameter at a time, keeping the others unchanged. The results (bending stresses and deflections) are given in terms of dimensionless equations linking all the independent parameters together as described earlier. In these equations, the bending stress is made non-dimensional by dividing it by the product of unit weight of grout mix and the width of the lining. The deflection is made dimensionless by expressing it in terms of the width of the lining. The non-dimensional bending stress (S/Gw) and deflection (δ/w) are plotted against $(w/t)^2$ and the lining flexibility, K, respectively, for staged grouting and flotation load, and against $(H/w)(w/t)^2$ and $(H/w)K$ for uniform pressure. From these plots, constants for the maximum bending stress and maximum deflection in the lining are computed for different boundary cases and different loading configurations. These are shown in Tables 1 and 2, respectively.

TABLE 1
Dimensionless Constants for the Maximum Bending Stress in the Lining

	Constant	*Boundary case 1*		*Boundary case 2*		*Boundary case 3*	
Staged grouting	A	0.3081	Node 26	−0.1118	Node 1	−0.0192	Node 1
Flotation	C	0.3401	Node 26	0.3437	Node 26	0.2033	Node 26
				0.0030	Node 1		
Uniform pressure	E	−0.7048	Node 26	−0.3938	Node 1	0.2940	Node 26
				0.0608	Node 26		

Note: positive values of A, C and E imply tensile stresses in the inner surfaces of the lining.

TABLE 2
Dimensionless Constants for the Maximum Deflections in the Lining

	Constants	*Boundary case 1*		*Boundary case 2*		*Boundary case 3*	
Staged	B_x	0.00000	Node 1	−0.00568	Node 16	0.00007	Node 4
grouting	B_y	0.06280	Node 1	−0.00447	Node 16	0.00026	Node 4
Flotation	D_x	0.00866	Node 13	0.00809	Node 13	0.00589	Node 15
		0.00813	Node 14	0.00390	Node 16	0.00572	Node 16
	D_y	−0.00216	Node 13	−0.00250	Node 13	0.00059	Node 15
		0.00223	Node 14	−0.00340	Node 16	0.00055	Node 16
Uniform	F_x	0.14770	Node 14	0.03354	Node 16	0.00639	Node 16
pressure		0.14188	Node 13	0.01959	Node 13	0.00594	Node 15
	F_y	0.09190	Node 14	0.02245	Node 16	0.00081	Node 16
		0.09110	Node 13	0.01981	Node 13	0.00069	Node 15

Note: inward deflections are taken as positive.

12 FULL-GROUTING DESIGN CURVES

12.1 Stress-limit criteria

Boundary case 1: restrained at crown only

For this case, Table 1 shows that the maximum bending stress in the lining resulting from each of flotation load and uniform pressure is located at the crown (i.e. at node 26) of the lining. Thus, using eqn (11) and appropriate constants from Table 1, the following design equation follows:

$$R = \left|0 \cdot 3401 - 0 \cdot 7048\left(\frac{p}{Gw} - 1 \cdot 5\right)\right| = \left|1 \cdot 3973 - 0 \cdot 7048\frac{p}{Gw}\right| \qquad (15)$$

It is further noticed that the maximum stresses developed at the crown, for these two loadings, are of opposite sense (in the case of flotation a compressive stress is developed at the outside of the lining, whereas in the case of uniform pressure loading the compressive stress develops at the inside of the lining). Hence, their combined effect on the lining will be less than the case in which the loads are applied separately. From similar considerations, the combined bending stresses have been critically examined at all other points and found to be less than at the crown.

Table 1 also shows that staged grouting is less critical than the flotation load alone. If staged grouting is employed, the relevant design information can readily be obtained from eqn (1) and Table 1.

It can be seen from Fig. 5 that, as one begins to grout, stresses increase first to a maximum (full flotation) before they decrease as additional head is superimposed (combined flotation and uniform pressure) up to the level of p/Gw equal to 2.465. Once the value of p/Gw exceeds 2.465, the combined effect of flotation and uniform pressure becomes more critical than flotation on its own.

Boundary case 2: restrained at crown and invert

It is clear from Table 1 that the maximum bending stress resulting from the flotation load is located at the crown of the lining (i.e. at node 26), whereas in the case of uniform-pressure load, the maximum bending stress is located at the invert of the lining (i.e. at node 1). This suggests that eqn (11) must be satisfied at both nodes 1 and 26 of the lining. The combined stresses at other nodes have been calculated and proved to be less critical than those at nodes 1 and 26.

The above leads to the following design expressions, which are shown graphically in Fig. 6. At node 26

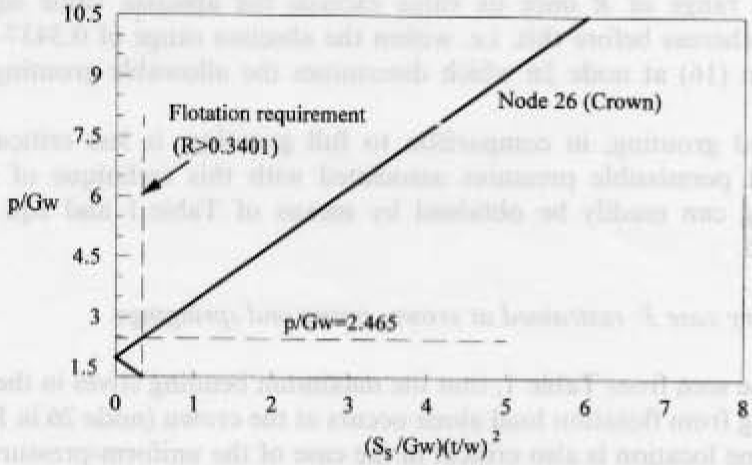

Fig. 5. Inverted egg-shaped lining: maximum bending stress at the crown of the lining for flotation ($p/Gw = 1.5$) and additional external pressure (boundary case 1).

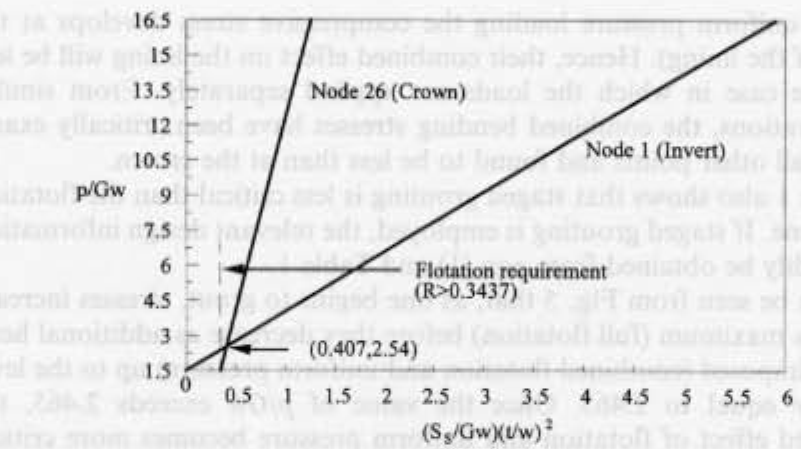

Fig. 6. Inverted egg-shaped lining: maximum bending stresses at the crown and invert of the lining for flotation ($p/Gw = 1.5$) and additional external pressure (boundary case 2).

$$R = \left|0.3437 + 0.0608\left(\frac{p}{Gw} - 1.5\right)\right| = \left|0.2525 + 0.0608\frac{p}{Gw}\right| \tag{16}$$

and at node 1

$$R = \left|0.0030 - 0.3938\left(\frac{p}{Gw} - 1.5\right)\right| = \left|0.5937 - 0.3938\frac{p}{Gw}\right|. \tag{17}$$

It emerges from Fig. 6 and Table 1 that a minimum value of R equal to 0.3437 is needed in order for the lining to withstand the maximum bending stress at the crown resulting from the flotation load alone.

It can also be shown from the figure that the allowable grouting pressure resulting from eqn (17) at node 1 becomes predominant throughout the full range of R once its value exceeds the abscissa value equal to 0.407, whereas before this, i.e. within the abscissa range of 0.3437–0.407, it is eqn (16) at node 26 which determines the allowable grouting pressure.

Staged grouting, in comparison to full grouting, is less critical. The relevant permissible pressures associated with this technique of staged grouting can readily be obtained by means of Table 1 and eqn (1), if required.

Boundary case 3: restrained at crown, invert and springings

It can be seen from Table 1, that the maximum bending stress in the lining resulting from flotation load alone occurs at the crown (node 26 in Fig. 4). The same location is also critical in the case of the uniform-pressure load. This simply means that eqn (11) is to be satisfied at node 26 (crown) of the lining, leading to the following design equation:

$$R = \left|0.2033 + 0.294\left(\frac{p}{Gw} - 1.5\right)\right| = \left|0.294\frac{p}{Gw} - 0.2377\right| \quad (18)$$

where

$$\frac{p}{Gw} > 1.5.$$

In this boundary case, as the pressure increases, the "criticality" increases.

Once again, as regards partial-grouting conditions (if adopted) for lining the sewer for this boundary case, the relevant data can readily be obtained by means of eqn (1) and Table 1.

Discussion on stress-limit criteria

Figure 7 provides a summary of the above three boundary conditions as applied to inverted egg-shaped sewer linings. Therefore, once a boundary case is selected and the geometrical and material parameters are chosen, a value of allowable grouting pressure based on the stress-limit criteria can be determined using Fig. 7. Alternatively, any of the variables appearing in Fig. 7 may be ascertained, provided all other variables are known.

It can easily be concluded from Fig. 7 that there is a finite range of abscissa values (0.3437–0.4223) for which boundary condition 2 is slightly more critical than boundary condition 1. It is also clear from the figure that, unlike the cut-off for boundary condition 1 at an abscissa value of 0.3401, the design curves for boundary conditions 2 and 3 have characteristics which reach the horizontal axis at values of 0.3437 and 0.2033, respectively. Boundary condition 3 is more critical than boundary case 1 within the abscissa range of 0.3401–0.592. Again, within the abscissa range of 0.3401–0.81, boundary condition 3 is more critical in comparison to boundary condition 2.

12.2 Deflection-limit criteria

Boundary case 1: restrained against flotation only

It is interesting to note from Table 2 that the maximum deflection in the lining resulting from staged grouting is greater than the one resulting from flotation load alone. This leads to the requirement that a minimum value of abscissa in Fig. 8 equal to 0.0628 is needed in order for the lining to withstand the maximum allowable deflection of 3% of w. Beyond this value, the full flotation and uniform pressure cases become critical.

In the case of flotation, the maximum deflection occurs at node 13, whereas such maximum occurs at node 14 for uniform pressure. This is

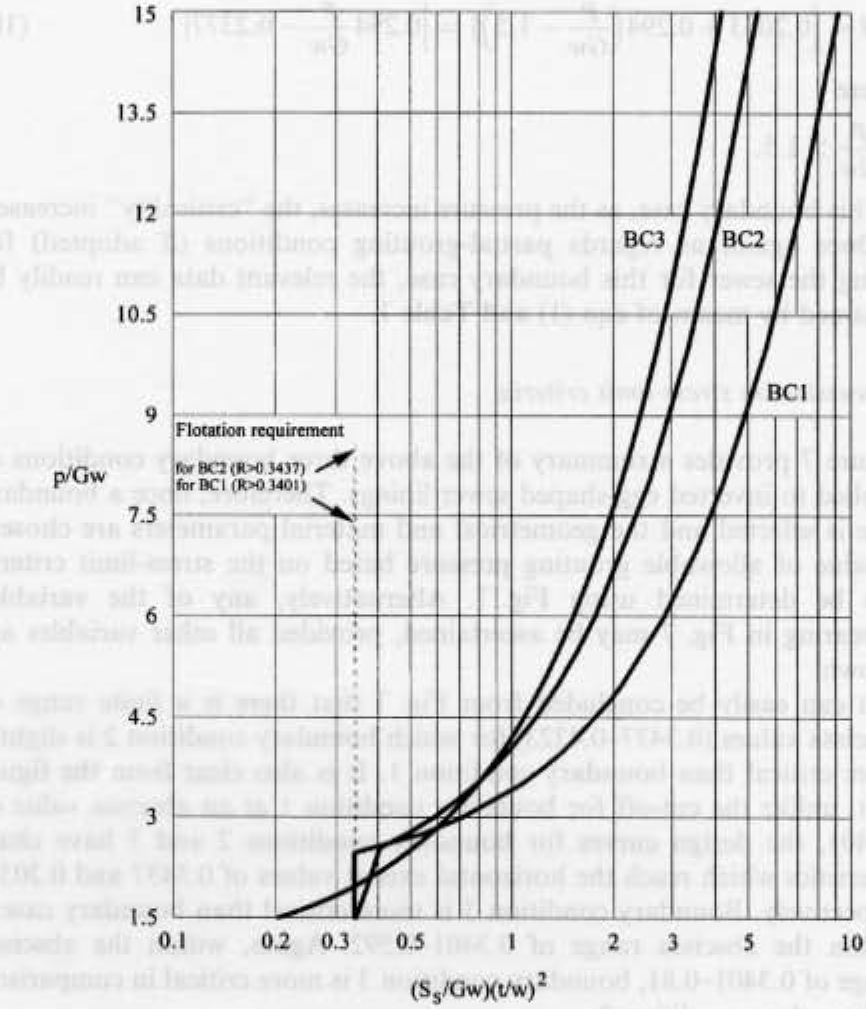

Fig. 7. Inverted egg-shaped lining: allowable grouting pressure for different boundary conditions, based on stress-limit criteria.

why eqn (13) must be satisfied at both the nodes, which gives rise to the following design expressions. At node 13

$$\frac{0.03}{K} = \left|0.169\frac{p}{Gw} - 0.224\right|. \quad (19)$$

At node 14

$$\frac{0.03}{K} = \left|0.174\frac{p}{Gw} - 0.25\right|. \quad (20)$$

By comparing these two equations, one finds that eqn (20) at node 14 is

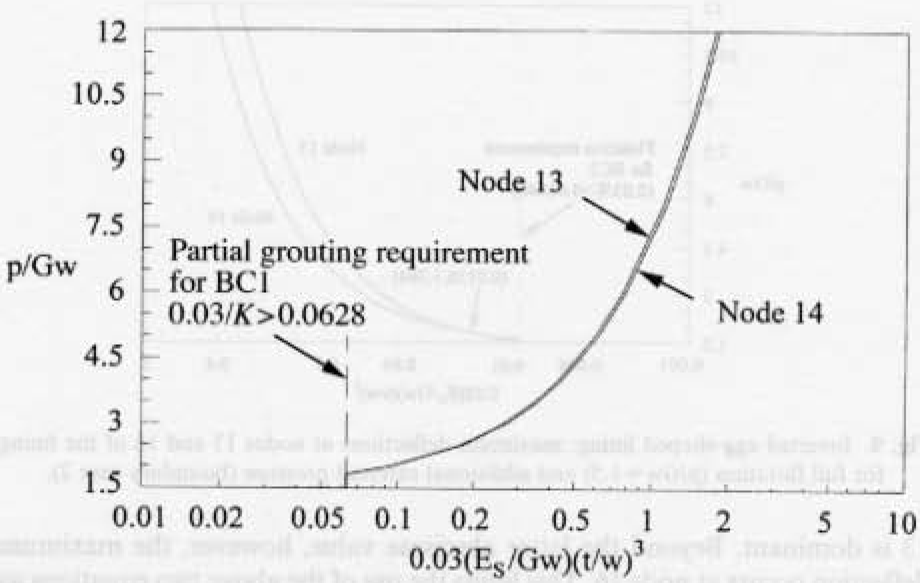

Fig. 8. Inverted egg-shaped lining: maximum deflections at nodes 13 and 14 of the lining for full flotation ($p/Gw = 1.5$) and additional external pressure (boundary case 1).

always critical, as combined displacement due to flotation and uniform pressure becomes always maximum at this node. This is illustrated in Fig. 8.

Boundary case 2: restrained at crown and invert

Table 2 shows that, unlike boundary condition 1, in the present case the maximum deflection in the lining resulting from staged grouting is less than that resulting from flotation load alone. This imposes a requirement that a minimum value of $0.03/K$ equal to 0.00846 is needed in order to withstand the maximum deflection of 3% of w. Once the value of $0.03/K$ exceeds 0.00846, the combined effect of flotation and uniform pressure becomes critical.

For the same reasons as were described under boundary condition 1, two design equations at nodes 13 and 16 are derived and are as follows. At node 13

$$\frac{0.03}{K} = \left|0.0279\frac{p}{Gw} - 0.0333\right|. \tag{21}$$

At node 16

$$\frac{0.03}{K} = \left|0.04036\frac{p}{Gw} - 0.0554\right|. \tag{22}$$

Equations (21) and (22) are shown graphically in Fig. 9. This clearly depicts that, for values of $0.03/K$ between 0.00846 and 0.0158, the deflection at node

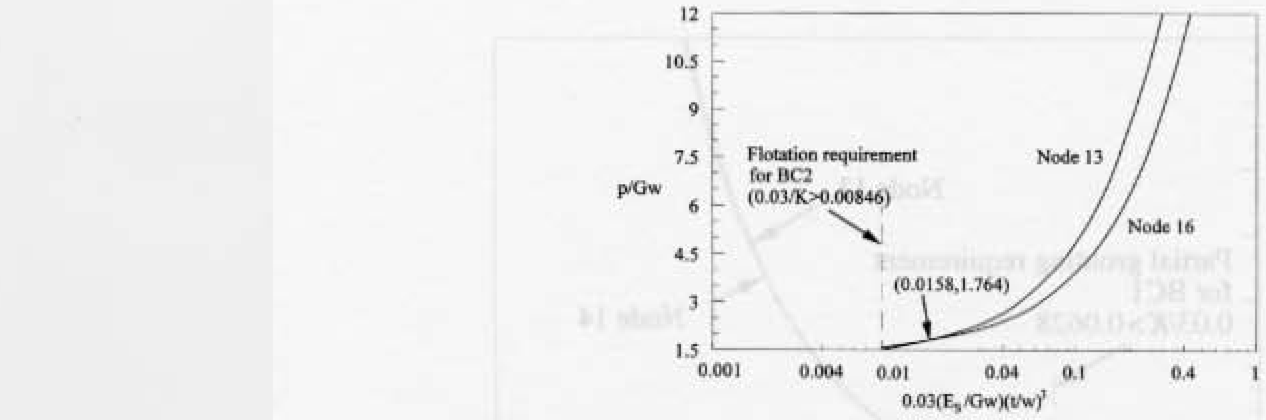

Fig. 9. Inverted egg-shaped lining: maximum deflections at nodes 13 and 16 of the lining for full flotation ($p/Gw = 1.5$) and additional external pressure (boundary case 2).

13 is dominant. Beyond the latter abscissae value, however, the maximum deflection occurs at node 16. This limits the use of the above two equations as follows: eqn (21) is valid for $0.5 \leq p/Gw \leq 1.764$ and eqn (22) is valid for $p/Gw \geq 1.764$, recalling, once again, that the above equations must always be satisfied as regards flotation loading, which implies that $0.03/K \geq 0.00846$. For this boundary case, the partial-grouting case has not been found to be critical. If required, because of the installation procedure adopted, the relevant information can be found by using the pertinent data of Table 2 and eqn (2).

Boundary case 3: restrained at crown, invert and springings

It is clear from Table 2 that, during staged grouting, the maximum deflection occurs at node 4 and its magnitude is much less than the flotation load as well as that from the combined load corresponding to flotation and uniform pressure. The maximum deflection in this boundary case due to the flotation load occurs at node 15, whereas such a peak is located at node 16 for uniform pressure. Hence, eqn (13) must be satisfied at both nodes 15 and 16 for the combined load of flotation and uniform pressure. This gives rise to the following two design equations. At node 15

$$\frac{0.03}{K} = \left|0.00598\frac{p}{Gw} - 0.00296\right|. \tag{23}$$

At node 16

$$\frac{0.03}{K} = \left|0.00644\frac{p}{Gw} - 0.00392\right|. \tag{24}$$

These two equations are plotted in Fig. 10. It emerges from Fig. 10 that 0.00592 is the minimum value of $0.03/K$ to withstand the maximum

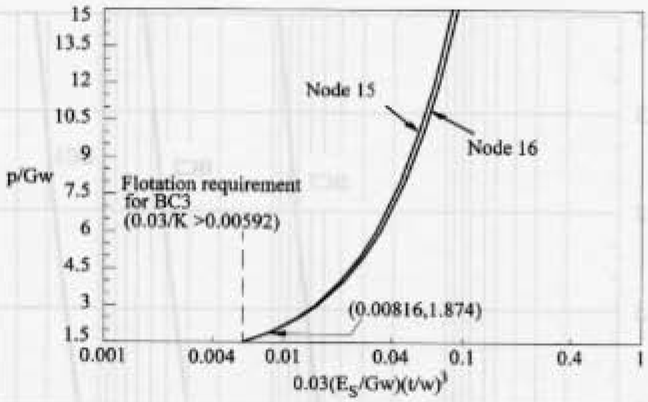

Fig. 10. Inverted egg-shaped lining: maximum deflections at nodes 15 and 16 of the lining for full flotation ($p/Gw = 1.5$) and additional external pressure (boundary case 3).

deflection at node 15 resulting from the flotation load. It is also shown that the allowable grouting pressure resulting from eqn (24) at node 16 becomes predominant once the value of $0.03/K$ exceeds 0.00816. Within the abscissa range of 0.00592–0.00816 the deflection at node 15 determines the design criteria, albeit by a very small margin.

Summary of deflection-limit criteria

Figure 11 summarizes the results of the above three boundary cases based on deflection-limit criteria and hence can be used to determine the allowable grouting pressure in any particular lining. It is found from the figure that, unlike the cut-off for boundary condition 1 at an abscissa value of 0.0602, boundary conditions 2 and 3 gradually reach the horizontal axis. It is also evident from the figure that, for a particular lining geometry and material property, boundary condition 3 gives a higher allowable pressure than boundary conditions 1 and 2. Boundary condition 2 also allows greater pressure to be withstood relative to boundary condition 1 during installation.

13 DISCUSSION OF PARAMETRIC ANALYSIS

13.1 Enhancement factors

The study of inverted egg-shaped linings has revealed how both maximum bending stress and maximum deflection in a lining resulting from grouting pressure can be reduced by introducing additional restraints during installation. This implies that an enhancement in the value of the grouting pressure can be achieved, thus providing adequate grouting of the annulus

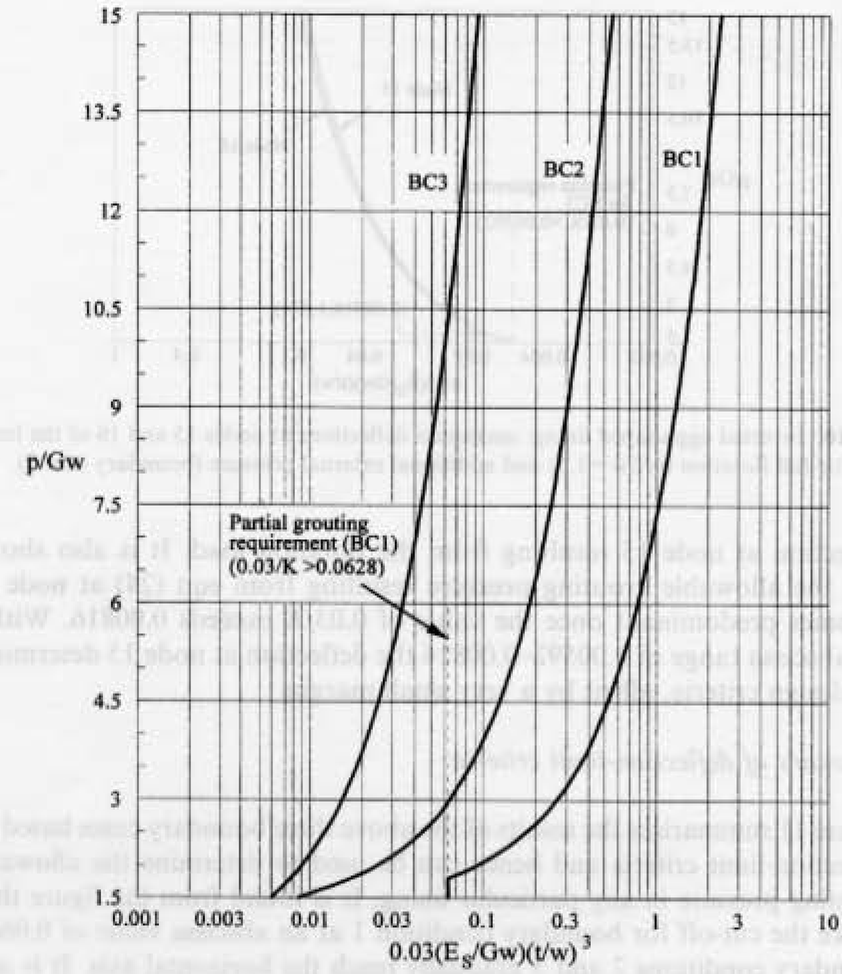

Fig. 11. Inverted egg-shaped lining: allowable grouting pressure for various boundary cases, based on deflection-limit criteria.

in addition to filling the voids in the neighbouring soil surround. This gives rise to the introduction of what can be termed an enhancement factor (EF). *The enhancement factor is defined as the ratio of the allowable grouting pressure which could be applied on any particular lining using boundary case 2 or 3 to the one corresponding to boundary case 1*, i.e.

$$EF_i = \frac{p_i}{p_1}. \tag{25}$$

Here $_i$ corresponds to boundary cases 2 or 3. For stress-limit criteria, p_1, p_2, p_3 can be calculated by using eqns (15)–(18), and they are given below:

$$p_1 = 1.42Gw(R + 1.3973) \tag{26a}$$

for $R > 0.3401$

$$p_2 = 16.45Gw(R - 0.2525) \tag{26b}$$

for $0.3437 < 0.4070$

$$p_2 = 2.54Gw(R + 0.5937) \tag{26c}$$

for $R > 0.4070$

$$p_3 = 3.4Gw(R + 0.2377) \tag{26d}$$

for $R > 0.2033$. With the help of eqns (25) and (26), enhancement factors, based on stress-limit criteria, are calculated for boundary cases 2 and 3 and plotted in Fig. 12. Similarly, by means of eqns (20)–(24), enhancement factors, based on deflection-limit criteria, were computed for boundary cases 2 and 3. This is shown pictorially in Fig. 13.

For a particular lining geometry and material properties, after calculating two enhancement factors [for both the stress- and deflection-limit criteria (from Figs 12 and 13)], the lower value of grouting pressure is to be taken in design. It is found from these figures that it is the stress limitation of the material which actually always governs the design calculations of the enhancement factor because this criterion results in a value of p/Gw which is much lower than the p/Gw value resulting from the deflection-limit criterion for boundary cases 2 and 3.

It is obvious from the figures that the EF increases as the short-term bending stress of the material (S_s) and the lining thickness (t) increase, while EF decreases with the increase of the width of the lining (w). From Fig. 12, one may also see that, at a value of R equal to 0.81, boundary conditions 2 and 3 provide the same enhancement factor. Once R exceeds

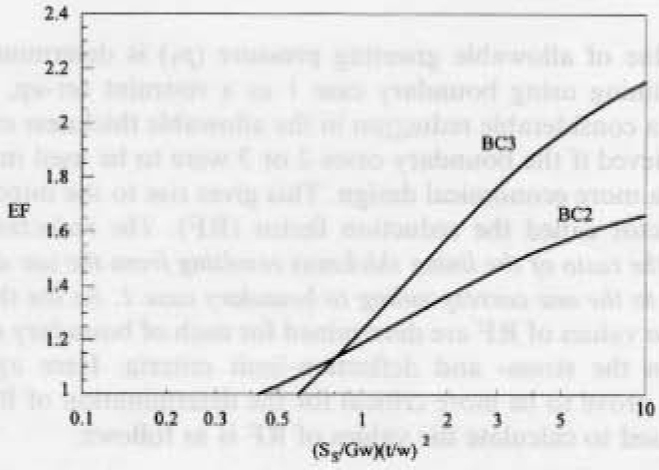

Fig. 12. Inverted egg-shaped lining: enhancement factor for allowable grouting pressure, based on stress-limit criteria.

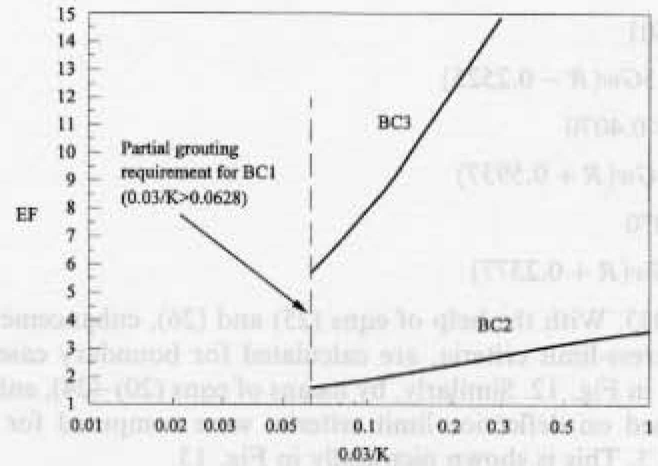

Fig. 13. Inverted egg-shaped lining: enhancement factor for allowable grouting pressure based on deflection-limit criteria.

0.81, boundary condition 3 always gives a higher value of enhancement factor than boundary condition 2.

The curves for boundary conditions 2 and 3 reach the abscissa at the values of 0.424 and 0.592, respectively. This means that, for boundary case 2, a minimum value of R greater than 0.424 is needed in order to attain a value of EF higher than one, implying that no beneficial effect can result from the use of boundary case 2 for a value of R of less than 0.424. Similar reasoning also applies to boundary condition 3, and in this case, the value of R must be greater than 0.592 to achieve an enhancement factor greater than unity.

13.2 Reduction factors

Once a value of allowable grouting pressure (p_1) is determined for any particular lining using boundary case 1 as a restraint set-up, it is to be noted that a considerable reduction in the allowable thickness of the lining can be achieved if the boundary cases 2 or 3 were to be used instead, thus leading to a more economical design. This gives rise to the introduction of another factor called the reduction factor (RF). *The reduction factor is defined as the ratio of the lining thickness resulting from the use of boundary case 2 or 3 to the one corresponding to boundary case 1.* As for the previous section, two values of RF are determined for each of boundary cases 2 and 3 based on the stress- and deflection-limit criteria. Here again, stress limitations prove to be more critical for the determination of RF, and the equation used to calculate the values of RF is as follows:

$$RF_i = \frac{t_i}{t_1} \tag{27}$$

where

$$t_i = [C_i + (\frac{p}{Gw} - 1.5)E_i]^{1/2}[Gw^3/S_s]^{1/2}$$

and

$$t_1 = [C_1 + (\frac{p}{Gw} - 1.5)E_1]^{1/2}[Gw^3/S_s]^{1/2}$$

with i corresponding to boundary cases 2 or 3, and other variables being defined by eqns (15)–(18) and Table 1.

The final equations (after some rearrangement of these expressions, so as to achieve a reduction factor of less than one) are given below:

$$(RF)_2 = \frac{t_2}{t_1} = \sqrt{\frac{0.5937 - 0.3938\frac{p}{Gw}}{1.3973 - 0.7048\frac{p}{Gw}}} \quad (28a)$$

$$(RF)_3 = \frac{t_3}{t_1} = \sqrt{\frac{0.2377 - 0.294\frac{p}{Gw}}{1.3973 - 0.7048\frac{p}{Gw}}} \quad (28b)$$

The above equations are shown pictorially in Fig. 14. It is seen from Fig. 14 that the reduction factor increases with the width of the lining and the specific gravity of the grout.

Figure 14 shows that, for boundary conditions 2 and 3, values of p_1/Gw greater than 2.580 and 2.823 are needed, respectively, to have reduction factors less than one. This, again, means that no beneficial effect can result from the use of boundary conditions 2 and 3 if the value of p_1/Gw is less than 2.58 and 2.823, respectively. Figure 14 also indicates that, at a value of p_1/Gw equal to 3.56, the reduction factor for both the boundary cases are the same (= 0.85). For p_1/Gw

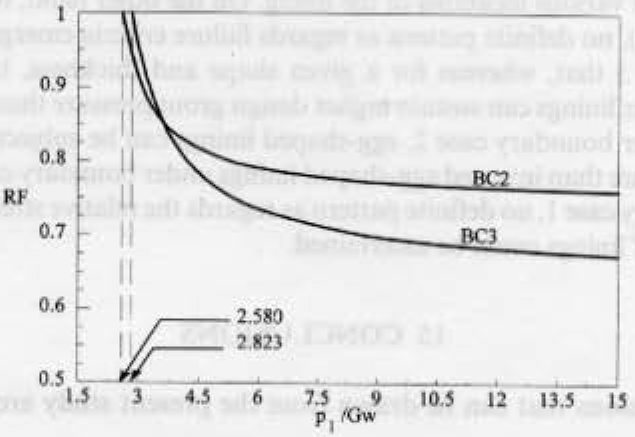

Fig. 14. Inverted egg-shaped lining: reduction factor for minimum permissible lining thickness based on stress-limit criteria.

greater than 3.56, boundary condition 3 provides higher reduction factors than boundary condition 2. The reverse is true for p_1/Gw less than 3.56.

14 COMPARATIVE STUDY OF THE STRUCTURAL PERFORMANCE OF INVERTED EGG-SHAPED LININGS WITH ITS EGG-SHAPED COUNTERPART

The geometry of egg-shaped sewer linings is exactly the same as that of inverted egg-shaped sewer linings (see Fig. 1), with the only exception being that its narrow end points downwards. An attempt has been made to compare the structural behaviour of egg- and inverted egg-shaped sewer linings, by varying several parameters and boundary conditions, with the aid of the design curves for inverted egg-shaped linings emanating from the present study and the design curves for egg-shaped linings available elsewhere.[1] Table 3 contains details of the findings of this study. In calculating the p/Gw values given in Table 3, R and $0.03/K$ are first calculated for various values of h, w and t. In this investigation, the value of G has been kept constant at $16.5 kN/m^3$. Again, in the comparative study GRP linings having E_s of $20\times10^6 kN/m^2$ and S_s of $60\times10^3 kN/m^2$ have been considered. Then, for inverted egg-shaped sewer linings, appropriate values of p/Gw are taken from Figs 7 and 11; the lower of the two being the governing value. It is to be noted that, in the event when the value of p/Gw falls out of the range of the given curves, appropriate values have been calculated using the relevant equations. A similar exercise has been performed for egg-shaped linings using the design curves given in Ref. [1]. Table 3 shows that, under boundary cases 2 and 3 (BC2, BC3), for both types of linings, the stress-limit criterion governs the design. The reason behind this may be attributed to the role of additional restraints on the reduction of deflection at various locations of the lining. On the other hand, for boundary case 1 (BC1), no definite pattern as regards failure criteria emerges. It is clear from Table 3 that, whereas for a given shape and thickness, inverted egg-shaped sewer linings can sustain higher design grout pressure than egg-shaped linings under boundary case 2, egg-shaped linings can be subjected to higher grout pressure than inverted egg-shaped linings under boundary case 3. Again, for boundary case 1, no definite pattern as regards the relative strength of these two types of linings could be ascertained.

15 CONCLUSIONS

The conclusions that can be drawn from the present study are as follows:

(4) It has been shown that, by introducing additional temporary restraints before grouting to the inverted egg-shaped sewer lining,

TABLE 3
Comparative Study of the Structural Capacity Between Egg-Shaped and Inverted Egg-Shaped Linings

Sewer lining no	h (mm)	w (mm)	t (mm)	Stress-limit criteria				Deflection-limit criteria				Governing criteria		
				R	p/Gw			0.03/K	p/Gw			BC1	BC2	BC3
					Boundary conditions 1	2	3		Boundary conditions 1	2	3			
Structural capacity of egg-shaped sewer linings														
1	1650	1100	15	0.794	2.13	2.13	3.73	0.118	1.89	2.95	16.90	D-L	S-L	S-L
2	1200	800	12	1.427	3.04	3.80	6.15	0.239	2.70	5.97	33.50	D-L	S-L	S-L
3	1650	1100	23	1.867	3.37	4.97	7.84	0.425	3.87	10.62	58.96	S-L	S-L	S-L
4	900	600	10	2.443	4.50	6.49	10.04	0.461	4.09	11.52	63.89	D-L	S-L	S-L
5	1200	800	20	3.963	6.68	10.51	15.86	1.107	8.16	27.67	152.00	S-L	S-L	S-L
6	900	600	15	5.496	8.88	14.57	21.74	1.555	10.97	38.87	213.00	S-L	S-L	S-L
Structural capacity of inverted egg-shaped sewer linings														
1	1650	1100	15	0.794	3.11	3.52	3.51	0.118	2.11	4.30	18.93	D-L	S-L	S-L
2	1200	800	12	1.427	4.00	5.13	5.66	0.239	2.81	7.29	37.70	D-L	S-L	S-L
3	1650	1100	23	1.867	4.63	6.24	7.16	0.425	3.88	11.90	66.60	D-L	S-L	S-L
4	900	600	10	2.443	5.44	7.71	9.12	0.461	4.10	12.80	72.20	D-L	S-L	S-L
5	1200	800	20	3.963	7.61	11.57	14.29	1.107	7.80	28.80	172.50	S-L	S-L	S-L
6	900	600	15	5.496	9.78	15.46	19.50	1.555	10.37	39.90	242.00	S-L	S-L	S-L

considerably higher grouting pressures, leading to a more reliable grouting operation, can be attained. The stress-limit criterion has been found to be the critical criterion in the design of linings with restraints. This stress-limit criterion also dictates the enhancement and reduction factors.

(5) Although, in the present study, permissible deflection has been taken as 3% of the width of the sewer lining, the proposed design curves can be adopted without any modification for any other allowable deflection-limit criteria set by the competent authority. (Only the abscissa's label changes by replacing the factor 0.03 by $n/100$ where n is the permissible deflection as percentage of the width.)

(6) The present study has concentrated on the short-term installation conditions corresponding to those linings that the WRC manual categorizes as Type I linings. These are defined as those linings which form a bond to the grout and/or the sewer wall, so that the renovated sewer acts as a composite section. However, the computations presented in this study are also relevant to sewer linings that do not form such a bond; structural improvement resulting from the strength of the lining itself. The WRC manual defines the latter as Type II linings. Their critical condition may be the long-term head of water, arising from infiltration through the old sewer, in which case the uniform-pressure loading and the boundary case 1 provide a close approximation to the analysis required.

(7) Throughout the present analyses, it has been assumed that all restraints are fully effective, so that the restrained points of the lining are prevented from moving in any direction. Such ideal conditions will very nearly be realized if internal supports are provided. On the other hand, external packing may not always be effective, in which case it might be necessary to assume that boundary condition 1 or 2 applies.

(8) A properly designed lining must be structurally sound for both the installation and in-service conditions. The loading of installation conditions differs from that of in-service conditions in many respects. During installation, the loading comes from grout pressure only. On the other hand, during service conditions grout pressure does not exist. It may happen that the lining is not bonded properly to the grout or the bond cannot be confidently relied upon. Hence, during service conditions, the grout may be considered to be cracked due to small ground movements, etc. If this happens, water may percolate through cracks and act at the interface between the lining and the grout. In this situation, a safe design head of water should be taken into consideration in the design of the lining structure.

(9) For the deflection-limit criteria chosen (3%w), inverted egg-shaped linings are structurally stronger than egg-shaped sewer linings under boundary case 2, whereas, under boundary case 3, egg-shaped sewer linings are stronger than inverted egg-shaped linings.

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