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Structural design of closely packed horseshoe-shaped sewer linings during installation

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Abstract

The paper describes the results of a numerical parametric study aimed at studying the structural response of closely packed horseshoe-shaped sewer linings. The effect of various restraint conditions which simulate different temporary support systems that may be used by the contractors during installation of the lining, and of different loading configurations which may arise at different stages of grouting the annulus gap between the lining and the sewer, have been thoroughly investigated. Covering the feasible range of geometric, material and loading parameters, comprehensive design curves — based on the allowable stress-limit, deflection-limit and (approximate) buckling criteria are presented. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Horseshoe-shaped sewer linings; Temporary supports; Grouting; Design criteria

Notation

A	dimensionless constants for maximum bending stress (staged grouting)
BC	boundary condition
B_x, B_y	dimensionless constants for maximum deflection (staged grouting)

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C	dimensionless constants for maximum bending stress (flotation)
D_x, D_y	dimensionless constants for maximum deflection (flotation)
E	dimensionless constants for maximum bending stress (uniform pressure)
E_s	short-term modulus of elasticity of lining material
EF	enhancement factor for allowable grouting pressure
F_{cr}	critical axial force in circular lining
F_x, F_y	dimensionless constants for maximum deflection (uniform pressure)
G	unit (specific) weight of grout mix
GRC	glass-reinforced cement
GRP	glass-reinforced plastic
H	excess head of grout (measured form crown of lining) corresponding to uniform pressure load
h	height of lining
K	$(Gw/E_s)(w/t)^3$
M	membrane stress at any point in lining
M_{cr}	critical buckling stress due to membrane action
M_t	total membrane stress
M_x	$D_x + F_x(H/w)$
M_y	$D_y + F_y(H/w)$
N_x	$D_x + F_x(p/Gw - h/w)$
N_y	$D_y + F_y(p/Gw - h/w)$
p	allowable grouting pressure measured at invert of lining
R	$(S_s/Gw)(t/w)^2$
RF	reduction factor for minimum permissible lining thickness
Q	$(360/\theta)^2 - 1$
S_s	allowable short-term bending stress of lining material
S_F	stiffness of lining, $(1/12)(E_s/(1 - \nu^2))(t/w)^3$
S_t	total bending stress of lining material due to combined flotation and external pressure
t	thickness of lining
WRC	Water Research Centre
w	width of lining
α	dimensionless constants for maximum membrane stress (flotation)
β	dimensionless constants for maximum membrane stress (uniform pressure)
δ	deflection of lining
δ_t	total deflection of lining material due to combined flotation and external pressure
θ	angle between supports of arch
ν	Poisson's ratio

1. Introduction

The large capital expenditure and well-known disruptions to traffic during the replacement of existing old sewers has drawn the engineers out of the traditional methods and led them to seek a better and easier solution — the lining of existing sewers. In the past, structural behaviour of circular-, egg- and inverted egg-shaped sewer linings under different installation and operational conditions have been investigated [1–6]. The present study concentrates on the behaviour of horseshoe-shaped sewer linings.

The linings are usually made of glass-reinforced plastic (GRP) or glass-reinforced cement (GRC). Steel linings are also used. Obviously, a horseshoe-shaped lining (see Fig. 1) is to be inserted into the similarly shaped sewer after allowing for an annulus gap so that the sewer lining fits within the existing sewer with a roughly uniform gap between the lining and the sewer walls. The gap between the lining and the sewer is then filled with a cementitious grout which, when set, creates a composite sewer-lining structure.

During installation, the lining is subjected to grout pressure. In some cases, this may lead to overstressing of the lining at different sections due to excessive bending moment, which may cause total collapse of the linings. Again, excessive deformation of any part of the lining might occur, affecting the serviceability of the relined sewer. Failure may also take place in the form of buckling due to excessive compressive forces. Thus, a properly designed sewer must comply to (bending) stress-limit, deflection-limit and buckling criteria. Here, the stress-limit criteria is so defined that the maximum bending stress developed during grouting must not exceed the allowable bending stress of the lining material. For deflection-limit criteria, a maximum allowable deflection in the lining not exceeding 3% of the width of the lining (as advocated by the Water Research Centre in its Sewerage Rehabilitation Manual [7]) has been followed. As regards buckling criteria, the lining must be so designed that failure is not triggered by buckling due to large hoop compression.

2. Grouting methods

During the installation of sewer linings, staged or partial grouting and full grouting techniques are generally adopted. Staged or partial grouting is performed in two stages. The first stage involves grouting the annulus up to the springings, and this is followed by a second stage carried out after the grout of stage one has set. On the other hand, full grouting is performed in a single stage. This technique is more practical than staged grouting. However, during full grouting, the lining is subjected to higher pressure so that a thicker lining or additional supports may be deemed essential in an effort to avoid excessive deformation or overstressing.

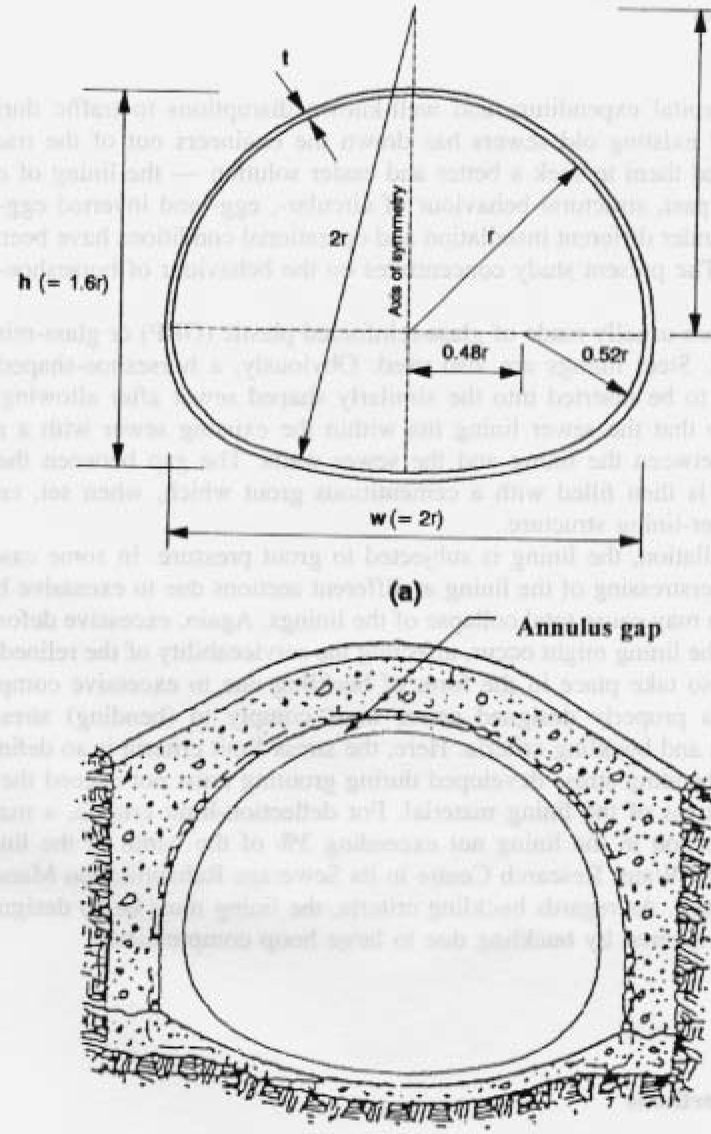


Fig. 1. Horseshoe-shaped lining: (a) shape of the lining adopted in the analysis and (b) the lining within the sewer.

3. Restraint conditions

The performance of egg-shaped, inverted egg-shaped and horseshoe-shaped linings is particularly sensitive to the type of support provided during grouting. Keeping this in mind, in the present study structural analyses of horseshoe-shaped linings have been carried out for three different support systems that may be used during installation. The support systems consist of hardwood wedges packed at different locations around the cross-section of the lining on the outside, together with internal struts positioned at the same locations. It is assumed that the packing between the sewer and the lining is closely spaced (typically, with spacing not exceeding the width of the lining), so that the structure can be studied by means of a two-dimensional finite-element (FE) model. The three possible support systems considered in the present study are shown in Fig. 2.

Boundary Condition 1 consists solely of a restraint at the crown (top) of the lining as shown in Fig. 2(a). It is to be noted here that grout is usually injected through the invert (bottom) of the lining. As grout moves forward and upward during its injection, this may push the lining upwards and, thereby, reduce the annulus gap

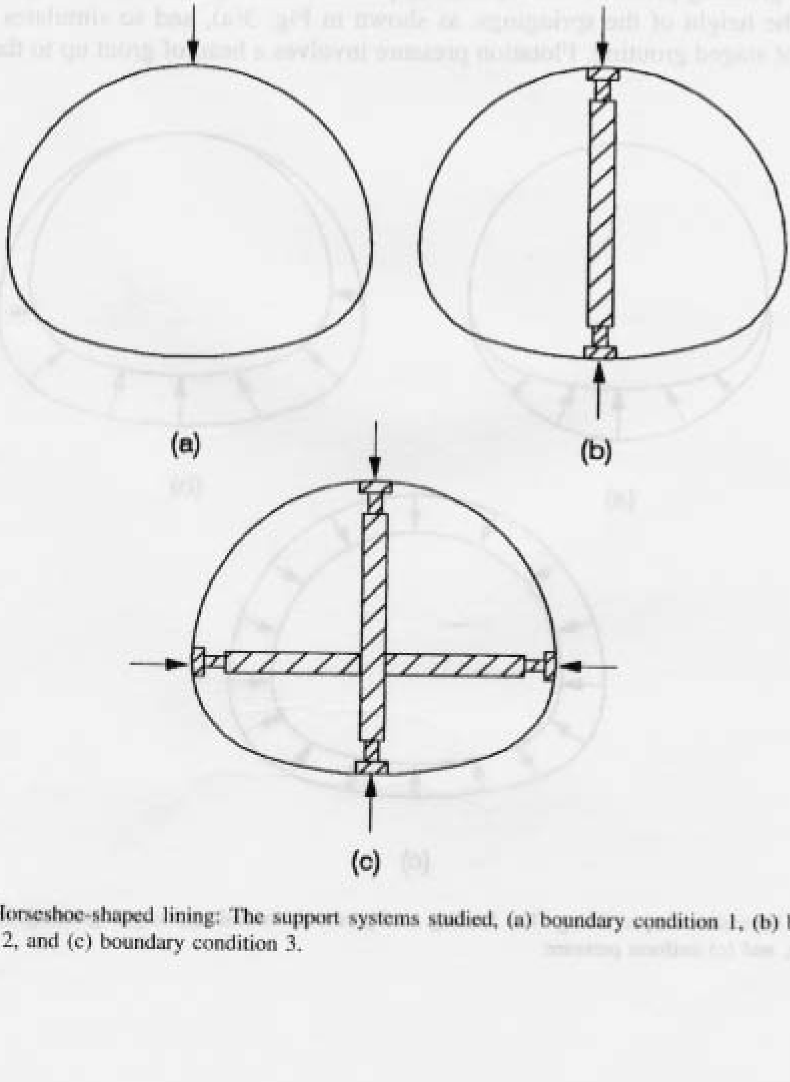


Fig. 2. Horseshoe-shaped lining: The support systems studied, (a) boundary condition 1, (b) boundary condition 2, and (c) boundary condition 3.

between the sewer and the lining. This is why a restraint at the crown is always expected. The second support system, shown in Fig. 2(b) as boundary condition 2, comprises restraints at both the crown and the invert of the lining. Like the boundary condition 1, boundary condition 2 imposes restraints on the vertical movement of sewer linings of not only the crown but also of the invert. Consequently, this boundary condition is vertically stiffer than the former. Boundary Condition 3 consists of restraints at the crown, invert and springings of the linings (Fig. 2(c)). In addition to vertical restraints, it restricts the horizontal movement of the lining at the springings.

4. Loading configurations

Three loading configurations, namely staged-grouting pressure, flotation pressure and uniform pressure are included throughout the analysis unless otherwise specified. Staged-grouting pressure corresponds to pressure from grout surrounding the lining up to the height of the springings, as shown in Fig. 3(a), and so simulates the first phase of staged grouting. Flotation pressure involves a head of grout up to the crown,

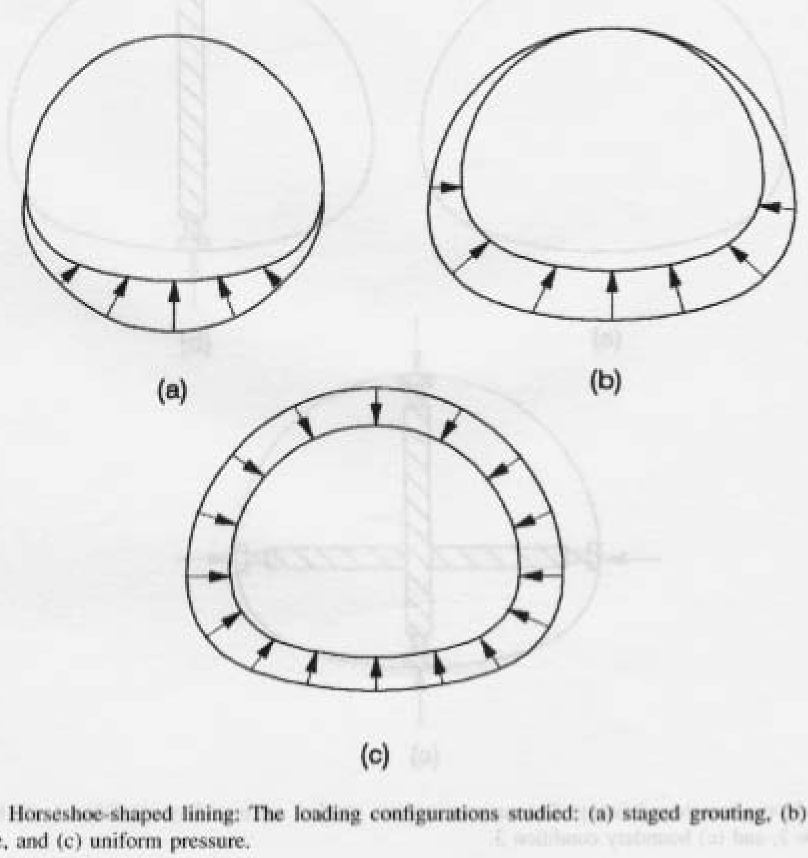


Fig. 3. Horseshoe-shaped lining: The loading configurations studied: (a) staged grouting, (b) flotation pressure, and (c) uniform pressure.

as in Fig. 3(b). In this situation, the lining is just covered by grout and hence the buoyancy force acting on the lining is the maximum that can occur. The uniform-pressure case, shown in Fig. 3(c), corresponds to the uniform pressure which is applied on the lining as a consequence of an excess head of grout. Clearly, flotation pressure and uniform pressure can be superimposed in order to simulate any grout pressure applied on the lining during full grouting.

5. Load calculation

In the case of the loading corresponding to flotation and to the first phase of staged grouting, the applied load is defined by the lining height h and the specific weight of grout mix G. In these two loading cases, the applied pressure at any point on the lining can be calculated by multiplying the specific weight of the grout mix by the distance from the top of the grouting to the point at which the pressure is calculated. For the uniform-load case, on the other hand, the external load is defined by the values of excess head of grout H (and its specific weight G) and is independent of the height of the lining.

6. Mathematical formulation of the analysis

For each load configuration and boundary case, a parametric study is carried out by varying one parameter at a time, keeping the others constant. The results are most conveniently given in terms of dimensionless equations linking all the independent parameters together. Such equations are derived on the basis of a curve-fitting exercise. Hence the design curves that will be proposed after an extensive parametric analysis of the horseshoe-shaped sewer linings can be used for all types of lining materials and lining sizes of that specific shape.

6.1. Dimensionless equations for bending stress and deflection

The dimensionless equations corresponding to the bending stress S and the deflection δ at any point on the lining can be written for the three load cases as follows:

1. Staged Grouting (Fig. 3(a))

$$S/Gw = A(w/t)^2 \quad (1)$$

$$\delta/w = (B_x^2 + B_y^2)^{1/2}K \quad (2)$$

2. Flotation (or pressure up to the level of crown) (Fig. 3(b))

$$S/Gw = C(w/t)^2 \quad (3)$$

$$\delta/w = (D_x^2 + D_y^2)^{1/2}K \tag{4}$$

3. Uniform Pressure (excess head H) (Fig. 3(c))

$$S/Gw = E(H/w)(w/t)^2 \tag{5}$$

$$\delta/w = (F_x^2 + F_y^2)^{1/2}(H/w)K \tag{6}$$

where $K = (Gw/E_s)(w/t)^3$ (7)

In these equations, S/Gw can be regarded as a non-dimensional stress while δ/w is the deflection related to the size of the lining and K is a measure of lining flexibility. Here, A, C, E, B_x, B_y, D_x, D_y, F_x and F_y are all constants which depend on the boundary set-up adopted during the grouting of the annulus and on the loading configuration used in the analysis.

The total bending stress S_t and the total deflection δ_t at any point in a lining subjected to a head of grout which is greater than the lining height h (i.e. full flotation) can be divided into values of bending stress and deflection resulting from the two loading cases of pressure up to the crown (i.e. flotation) and uniform pressure. This implies that, by adding Eqs. (3) and (5), and Eqs. (4) and (6), the following dimensionless equations for the total bending stress and the total deflection, respectively, can be written as

$$S_t/Gw = |(C + E(H/w))(w/t)^2| \tag{8}$$

$$\delta_t/w = (M_x^2 + M_y^2)^{1/2}K \tag{9}$$

where

$$M_x = D_x + F_x(H/w) \tag{10a}$$

$$M_y = D_y + F_y(H/w) \tag{10b}$$

Since, as mentioned earlier, the maximum bending stress and the maximum deflection in a lining must not exceed the respective values of S_s and *0.03 w*, the values of S_t and δ_t in Eqs. (8) and (9) can be replaced by S_s and $0.03w$, respectively. As the point of injection of the grout is usually located at the invert of the lining, it is convenient to replace the value of H in Eqs. (8) and (9) by the equivalent expression $(p/G - h)$, where p is the allowable grouting pressure measured at the invert of the lining. As a result, Eqs. (8) and (9) can be rewritten to produce the following design equations

$$R = |C + E(p/Gw - h/w)| \tag{11}$$

where,

$$R = (S_t/Gw)(t/w)^2 \tag{12}$$

and

$$0.03/K = (N_x^2 + N_y^2)^{1/2} \tag{13}$$

where,

$$N_x = D_x + F_x(p/Gw - h/w) \tag{14a}$$

$$N_y = D_y + F_y(p/Gw - h/w) \tag{14b}$$

6.2. Dimensionless equations for membrane stress

Although buckling is unlikely to be the governing criterion in horseshoe-shaped linings, if adequate temporary restraints are provided during their installation, the fact that axial forces in these linings are of comparable magnitudes to those in circular ones (namely, of the order of external pressure × radius, whereas in egg-shaped linings the axial forces are only 10%–40% (depending on boundary restraints) of such magnitude) suggests that stability considerations should not be neglected altogether. In order to consider buckling — albeit approximately — in the analysis, the simplified approach in [5] for circular linings has been followed. It has been found that dimensionless Eqs. (15) and (16) below, which had been found suitable for circular linings [5] under flotation and uniform pressure cases, respectively, are equally applicable to horseshoe-shaped linings once w is inserted instead of D. Here, M corresponds to the membrane stress at any point in the lining.

1. Flotation

$$M/Gw = \alpha(w/t) \tag{15}$$

2. Uniform Pressure

$$M/Gw = \beta(w/t)(H/w) \tag{16}$$

where α and β are constants which depend on the boundary case selected during installation.

The total direct membrane stress (M_t) at any point in a lining subjected to a head of grout which is greater than the lining height h (i.e. full grouting), can be obtained by adding the values of membrane stresses resulting from each of the flotation loading and the uniform pressure. This leads to the following dimensionless equation for the total membrane stress in the lining:

$$(M_t/Gw)(t/w) = (\alpha + \beta(H/w)) \tag{17}$$

In Eq. (17), the replacing of the value of H by the equivalent expression $(p/G - h)$ and of h/w by 0.8, produces the following generic design equation for the different boundary conditions:

$$(M_t/Gw)(t/w) = \alpha + \beta(p/Gw - 0.8) \tag{18}$$

Here, the value of the critical grouting pressure, that can be applied on the lining during its installation, is based on a direct stress-limit criteria which is equal to the critical buckling stress due to membrane action M_{cr} in a hinged arch of equivalent radius and unrestrained length [8]. The value of M_{cr} is given by the following equation:

$$(M_{cr}/Gw)(t/w) = 4.0Q(S_F/Gw) \tag{19}$$

where S_F is the stiffness of the lining which is given as follows:

$$S_F = (1/12)(E_t/(1 - \nu^2))(t/w)^3 \tag{20}$$

Q in Eq. (19) is a constant which depends on the angle θ between the hinges of the arch and is expressed as follows:

$$Q = (360°/\theta)^2 - 1 \tag{21}$$

so that Q takes on the values 3 and 15 for boundary cases 2 and 3, respectively.

By equating expressions 18 and 19 and using the appropriate value of Q from Eq. (21), a general design equation for the critical buckling pressure can be derived, albeit approximately, as follows:

$$4Q(S_F/Gw) = \alpha + \beta(p/Gw - 0.8) \tag{22}$$

The approximation implicit in this simplified buckling criterion is the adoption of hinges at the restrained points (thus neglecting continuity) [5] and in approximating the effective arc by an equivalent circular one.

6.3. Design criteria

For any particular lining geometry and material properties, the above equations must be satisfied at the locations of maximum bending stress, deflection and axial stress in the lining. The maximum allowable grouting pressure p which can be applied on the lining during grouting is the minimum of the p values as determined by all the criteria described in the previous two sections.

7. Two-dimensional finite-element model

A linear two-dimensional FE model is used in order to simulate the behaviour of horseshoe-shaped linings under various probable loads during installation. It can be seen from the shape of the horseshoe-shaped sewer of Fig. 1 that its bottom corners have sharp bends. Since higher concentrated stresses are expected at, or close to, these sharp bends, these stress concentrations may be catered for by using a thicker liner at the corners or by smoothening the corners by employing circular arcs. In the present study, the corners of the sewer lining have been given a slightly different geometry than that of the actual sewer. While the shape of the lining adopted for the analysis has been shown in Fig. 1(a), the actual lining geometry, in conjunction with the horseshoe-shaped sewer, is given in Fig. 1(b). It is clear that the height of the lining h is 0.8 times its width (i.e. $h/w = 0.8$); also, the annulus gap between the sewer and the lining becomes non-uniform, with a slightly higher gap, near the bends.

In the analysis, the thickness of the lining is assumed to be constant all around the cross-section. Due to symmetry of the lining geometry, loading and boundary conditions about the vertical axis (i.e. Y-axis), only half of the cross-section, shown in Fig. 4, is analysed. The elements used in the analysis are two-noded beam elements each having three degrees of freedom (horizontal and vertical displacement, and rotation) at each node. The mesh adopted consists of 30 elements, the node numbers corresponding to crown, springing and invert being 31, 14 and 1, respectively.

The restraints due to the support system shown in Fig. 2 are simulated numerically in the analysis by fixing the horizontal and vertical components of displacement at the corresponding nodal points. This involves a small approximation in that the deformation in the restraining struts is ignored, the strut being very stiff compared with the lining. As half of the cross-section is analysed, the horizontal and rotational components of displacements at nodes 1 and 31 of the lining are fully restrained. In addition, in Fig. 4, the vertical displacement at node 31 is set to zero for boundary case 1 while the vertical displacements at nodes 1 and 31 are made equal to zero for boundary case 2. Similarly, restraints have been imposed on the vertical displacement at node 1 and 31, and on vertical and horizontal displacements at node 14 (springing) in order to simulate boundary case 3.

The various loading configurations shown in Fig. 3 have been simulated by applying equivalent point loads at appropriate nodes.

8. Computation of constants

As already explained, for each load and boundary case, the parametric analysis is carried out by varying one parameter at a time, keeping the others unchanged. The results (bending stresses, deflections and axial stresses) are given in terms of dimensionless equations linking all the independent parameters together as described earlier. The non-dimensional bending stress (S/Gw) and deflection (δ/w) are plotted against $(w/t)^2$ and lining flexibility K, respectively for staged grouting and flotation load, and against $(H/w)(w/t)^2$ and $(H/w)K$ for uniform pressure. Similarly, the non-

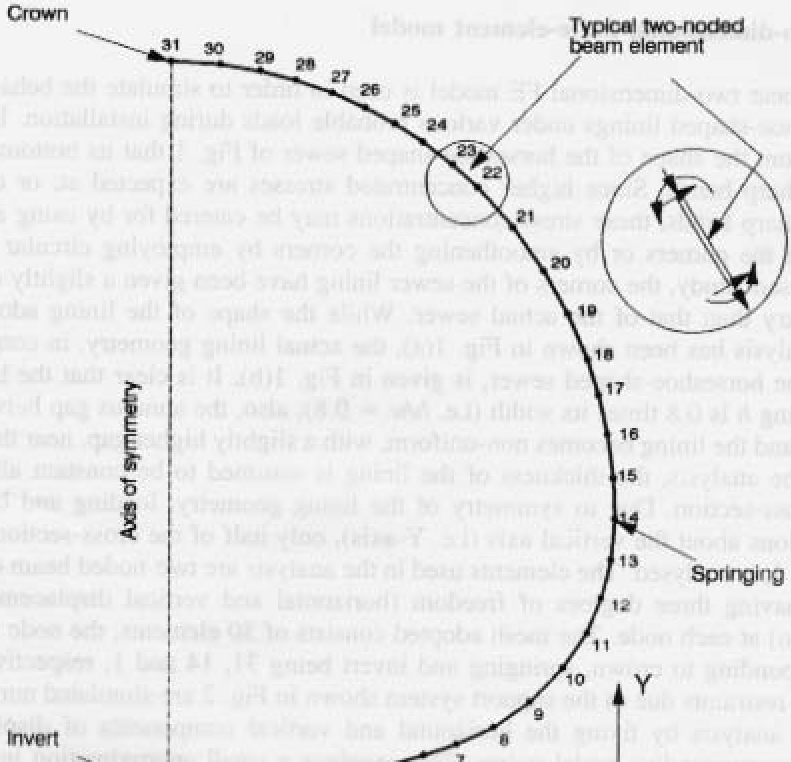


Fig. 4. Horseshoe-shaped lining: Two-dimensional FE mesh adopted in the analysis.

dimensional membrane stress (M/Gw) is plotted against (w/t) and $(w/t)(H/w)$, respectively for flotation and uniform pressure cases. From these plots, constants for the maximum bending stress, maximum deflection and maximum membrane stress in the lining are computed for different boundary cases and different loading configurations. These are listed in Tables 1–3, respectively.

9. Full-grouting design curves

9.1. Stress-limit criteria

9.1.1. Boundary condition 1: restrained at crown against flotation

The values of the bending-stress constants A, C, and E for this boundary case for different loading conditions are shown in Table 1. It is seen from these values that the absolute magnitude of C (flotation-load case) at node 31 (at the crown) is the

Table 1
Dimensionless constants for the maximum bending stress in the lining

	Constant	Boundary Case 1	Boundary Case 2	Boundary Case 3
Staged grouting	A	0.1956 Node 31	− 0.0674 Node 1	− 0.0204 Node 1
Flotation	C	0.4482 Node 31	− 0.1381 Node 1	− 0.0535 Node 1
		0.2477 Node 1	0.1063 Node 31	
Uniform pressure	E	0.1961 Node 1	− 0.1027 Node 31	− 0.0611 Node 1
		0.0783 Node 31	− 0.0071 Node 1	

(Note: positive values of A, C and E imply tensile stresses in the inner surfaces of the lining)

Table 2
Dimensionless constants for the maximum deflection in the lining

	Coefficient	Boundary case 1	Boundary case 2	Boundary case 3
Staged grouting	B_x	0.00000 Node 1	− 0.00120 Node 19	0.00003 Node 5
	B_Y	0.03610 Node 1	− 0.00213 Node 19	0.00242 Node 5
Flotation	D_x	0.00000 Node 1	0.00219 Node 20	0.00008 Node 5
	D_Y	0.07722 Node 1	0.00466 Node 20	0.00065 Node 5
Uniform pressure	F_x	0.00000 Node 1	0.00230 Node 20	0.00008 Node 5
	F_Y	0.04071 Node 1	0.00485 Node 20	0.00072 Node 5

(Note: Inward deflections are taken as positive)

Table 3
Dimensionless constants for the maximum direct membrane stress in the lining

	Constant	Boundary case 2	Boundary case 3
Flotation	α	0.19 node 21	0.27 Node 10
		0.16 node 10	0.10 Node 24
Uniform pressure	β	0.37 node 21	0.38 Node 10
		0.42 node 10	0.51 Node 24

greatest of all the five values tabulated in the column. This means that a minimum value of R equal to 0.4482 is needed in order for the lining to withstand the maximum bending stress at the crown resulting from the flotation load alone.

It can be seen from the table that, whereas in case of the flotation load the maximum bending stress occurred at node 31, that for uniform pressure is located at node 1. This suggests that, for the case of full grouting (flotation plus uniform pressure), Eq. (11) must be satisfied at both nodes 31 and 1 of the lining. This is why the values of C and E are computed at nodes 1 and 31, thus ensuring that the worst case for the full-grouting load can be designed for. In this connection, it should be stated that the combined bending stresses at all other nodes were also computed and were found to be less critical than those at nodes 1 and 31. The above discussion leads to the following design equations which are shown graphically in Fig. 5(a).

At node 31,

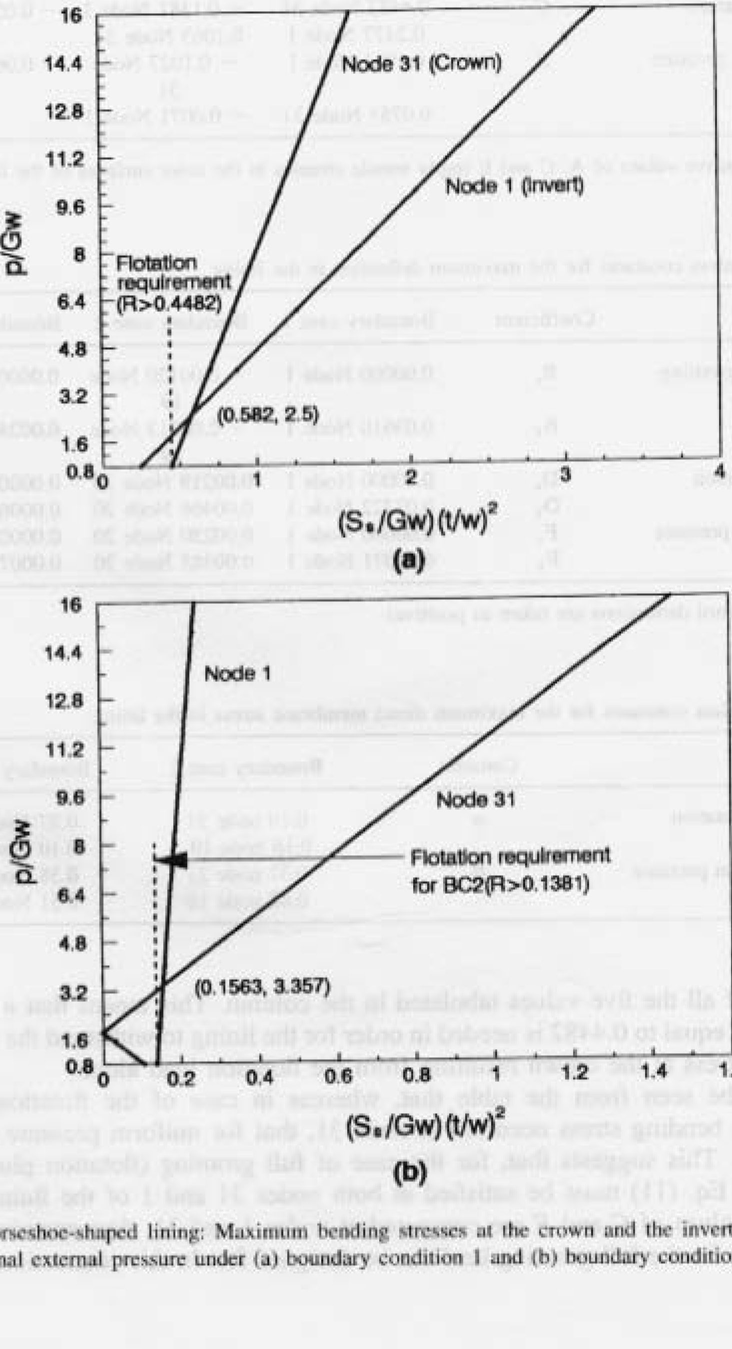


Fig. 5. Horseshoe-shaped lining: Maximum bending stresses at the crown and the invert for flotation and additional external pressure under (a) boundary condition 1 and (b) boundary condition 2.

$$R = |0.4482 + 0.0783(p/Gw - 0.8)| = |0.3856 + 0.0783p/Gw| \quad (23)$$

At node 1,

$$R = |0.2477 + 0.1961(p/Gw - 0.8)| = |0.0908 + 0.1961p/Gw| \quad (24)$$

It emerges from Fig. 5(a) that the allowable grouting pressure resulting from Eq. (23) remained critical with the value of R between 0.4482 and 0.5820; once its value exceeds 0.5820, Eq. (24) becomes dominant since the bending stress at node 1 then becomes critical.

Staged grouting is not critical in comparison to full grouting. If needed, the relevant permissible grouting pressures associated with this technique can be obtained by means of Table 1 and Eq. (1).

9.1.2. Boundary condition 2: restrained at crown and invert of the lining

For this boundary case, the relevant bending-stress constants for the different loading configurations are given in Table 1. It is seen from the table that the maximum bending-stress resulting from the flotation load is located at the invert of the lining (i.e. at node 1), whereas in the case of the uniform-pressure load, the maximum bending-stress location is at the crown (i.e. at node 31) of the lining. This implies that Eq. (11) is to be satisfied at both these nodal points. The combined bending stresses at the other nodes were calculated and proved to be less critical than those at nodes 1 and 31. This leads to the following two design expressions:

At node 1,

$$R = |-0.1381 - 0.0071(p/Gw - 0.8)| = |-0.1324 - 0.0071p/Gw| \quad (25)$$

At node 31,

$$R = |0.1063 - 0.1027(p/Gw - 0.8)| = |0.1885 - 0.1027p/Gw| \quad (26)$$

These two equations are shown pictorially in Fig. 5(b). It emerges from Fig. 5(b) and Table 1 that a minimum value of R equal to 0.1381 is required to take care of the maximum bending stress developed in horseshoe-shaped lining during the application of the flotation loading.

It is seen from the figure that, within the range of R values 0.1381 and 0.1563, the stress at node 1, i.e. the invert, is critical. However, beyond the value of R equal to 0.1563, the maximum bending stress occurs at node 31, i.e. at the crown of the lining.

As for the boundary condition 1, partial grouting is less critical than the other two types of loadings in the present boundary case 2. If staged grouting is employed during the installation of the sewer lining, the allowable grouting pressure can readily be determined in a manner similar to boundary condition 1.

9.1.3. Boundary condition 3: restrained at crown, invert and springing of the lining

In this boundary case, the maximum bending stress is located at the invert of the lining for both loading cases. Hence, at node 1, the following design expression can be written through the use of Eq. (1):

$$R = |-0.0535 - 0.0611(p/Gw - 0.8)| \quad (27)$$

$$= |-0.0046 - 0.0611p/Gw|$$

with $p/Gw \geq 0.8$, because as for the boundary cases 1 and 2, a full head of grout must be imposed on the lining for the critical condition to be realised. Once again, if partial grouting conditions are required, they can be found by means of Eq. (1) and Table 1.

9.1.4. Summary of stress-limit criteria

All the findings and conclusions described earlier for the three boundary cases studied in the context of full grouting are summarised in Fig. 6. From this figure, the allowable grouting pressure on a particular lining for any boundary case, based on the stress-limit criteria, can be obtained, if the geometrical and material parameters of the lining are known.

It is seen from Fig. 6 that, for a given value of R, boundary condition 2 gives allowable grouting pressures higher than those of boundary condition 1. Similarly, boundary case 3 provides higher allowable pressures than the other two boundary conditions (except for a small range of R in which boundary condition 2 gives the greater value of allowable grouting pressure). Hence, as expected, increasing restraints permit higher grouting pressure to be applied.

9.2. Deflection-limit criteria

9.2.1. Boundary condition 1: restrained at crown only

For this boundary case, Table 2 shows that the maximum deflection in the lining, resulting from each of the flotation and uniform-pressure loading cases, is located at the invert of the lining (at node 1). This means that Eq. (13) must be satisfied at node 1 of the lining, leading to the following design equation:

$$0.03/K = |0.04071p/Gw + 0.04465| \quad (28)$$

where $p/Gw > 0.8$ holds, because in this boundary case flotation rather than partial grouting is critical. If needed, the latter case can be calculated using Table 2 and Eq. (2).

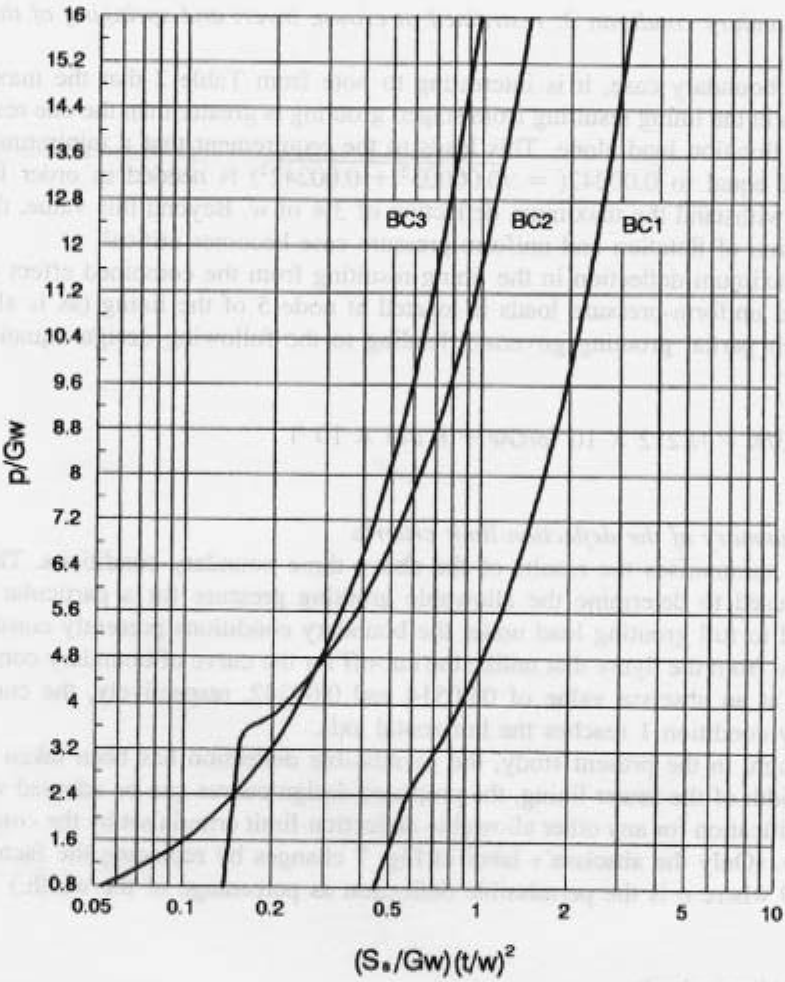


Fig. 6. Horseshoe-shaped lining: Allowable grouting pressure, based on stress-limit criteria, for various boundary conditions.

9.2.2. Boundary condition 2: restrained at crown and invert

From Table 2, it is seen that the maximum deflection for staged grouting, flotation and uniform-pressure loading occurs at nodes 19, 20 and 20, respectively. Hence, for full grouting (flotation plus uniform pressure), the maximum deflection will occur at node 20. By using Eq. (13), the following design equation can be deduced at this node of maximum displacement

$$0.03/K = |0.00537p/Gw - 0.00944| \tag{29}$$

for $p/G \geq h$. Here, partial grouting is not critical in comparison to flotation or full grouting.

9.2.3. Boundary condition 3: restrained at crown, invert and springing of the lining

In this boundary case, it is interesting to note from Table 2 that the maximum deflection in the lining resulting from staged grouting is greater than the one resulting from the flotation load alone. This leads to the requirement that a minimum value of *0.03/K* equal to 0.00242$(= \sqrt{0.00003^2 + 0.00242^2})$ is needed in order for the lining to withstand the maximum deflection of 3% of w. Beyond this value, the full combination of flotation and uniform-pressure case becomes critical.

The maximum deflection in the lining resulting from the combined effect of flotation and uniform-pressure loads is located at node 5 of the lining (as is also the case when partial grouting governs), leading to the following design equation for $p/G \geq h$:

$$0.03/K = |7.212 \times 10^{-4} p/Gw + 8.144 \times 10^{-5}| \tag{30}$$

9.2.4. Summary of the deflection-limit criteria

Fig. 7 summarises the results of the above three boundary conditions. This can now be used to determine the allowable grouting pressure for a particular lining subjected to full grouting load under the boundary conditions presently considered. It is noted from the figure that unlike the cut-off for the curve of boundary conditions 2 and 3 at an abscissa value of 0.00514 and 0.00242, respectively, the curve for boundary condition 1 reaches the horizontal axis.

Although, in the present study, the permissible deflection has been taken as 3% of the width of the sewer lining, the proposed design curves can be adopted without any modification for any other allowable deflection-limit criteria set by the competent authority. (Only the abscissa's label in Fig. 7 changes by replacing the factor 0.03 by n/100 where n is the permissible deflection as percentage of the width.)

9.3. Buckling criteria

9.3.1. Boundary condition 2: restrained at crown and invert

The dimensionless constants α and β for the maximum direct membrane stresses in the lining are listed in Table 3. For boundary condition 2, the maximum membrane stress develops at node 21 in the case of the flotation load, whereas under uniform-pressure load the maximum stress occurs at node 10. Thus, Eq. (22) must be satisfied at both nodes 10 and 21. This leads to the following two design expressions:

At node 10,

$$\begin{aligned} 4Q(S_F/Gw) &= 0.16 + 0.42(p/Gw - 0.8) \\ &= 0.42p/Gw - 0.176 \end{aligned} \tag{31}$$

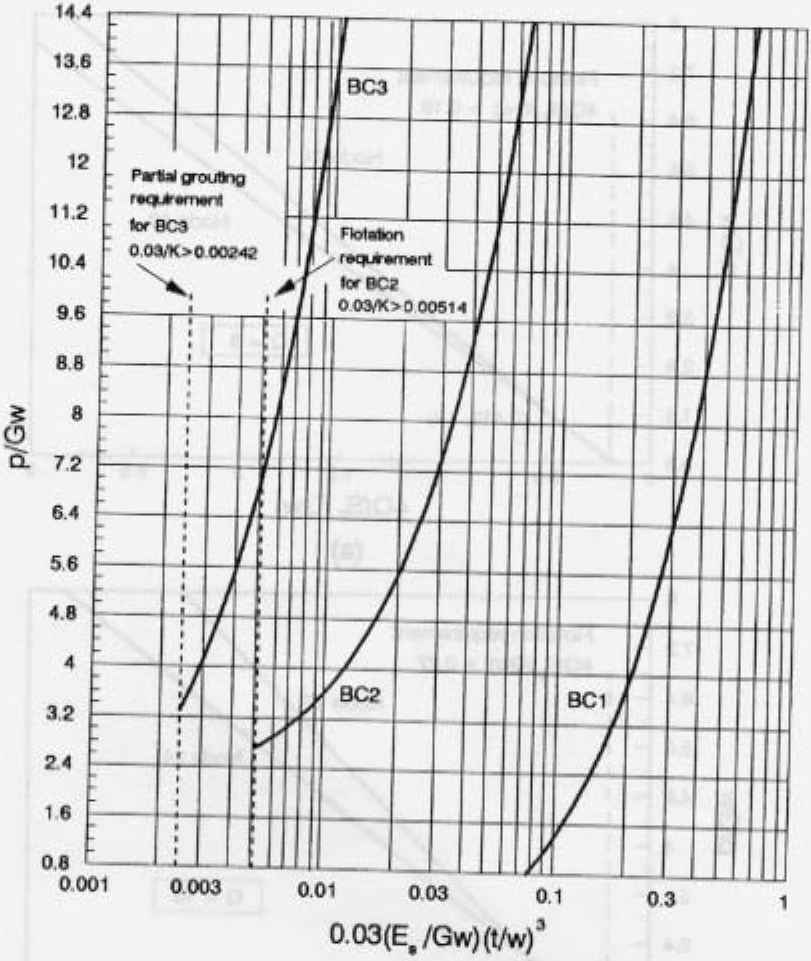


Fig. 7. Horseshoe-shaped lining: Allowable grouting pressure based on deflection-limit criteria, for various boundary conditions.

At node 21,

$$\begin{aligned} 4Q(S_r/Gw) &= 0.19 + 0.37(p/Gw - 0.8) \\ &= 0.37p/Gw - 0.106 \end{aligned} \tag{32}$$

Eqs. (31) and (32) are shown pictorially in Fig. 8(a). From this figure it is clear that, up to a value of p/Gw equal to 1.4 (i.e. $4Q(S_r/Gw)$ equal to 0.412), Eq. (32) is valid while, beyond this point, Eq. (31) dictates the allowable grouting pressure. (Clearly, the combined membrane stresses at all other nodes were computed and were found to be less critical than those at nodes 10 and 21.)

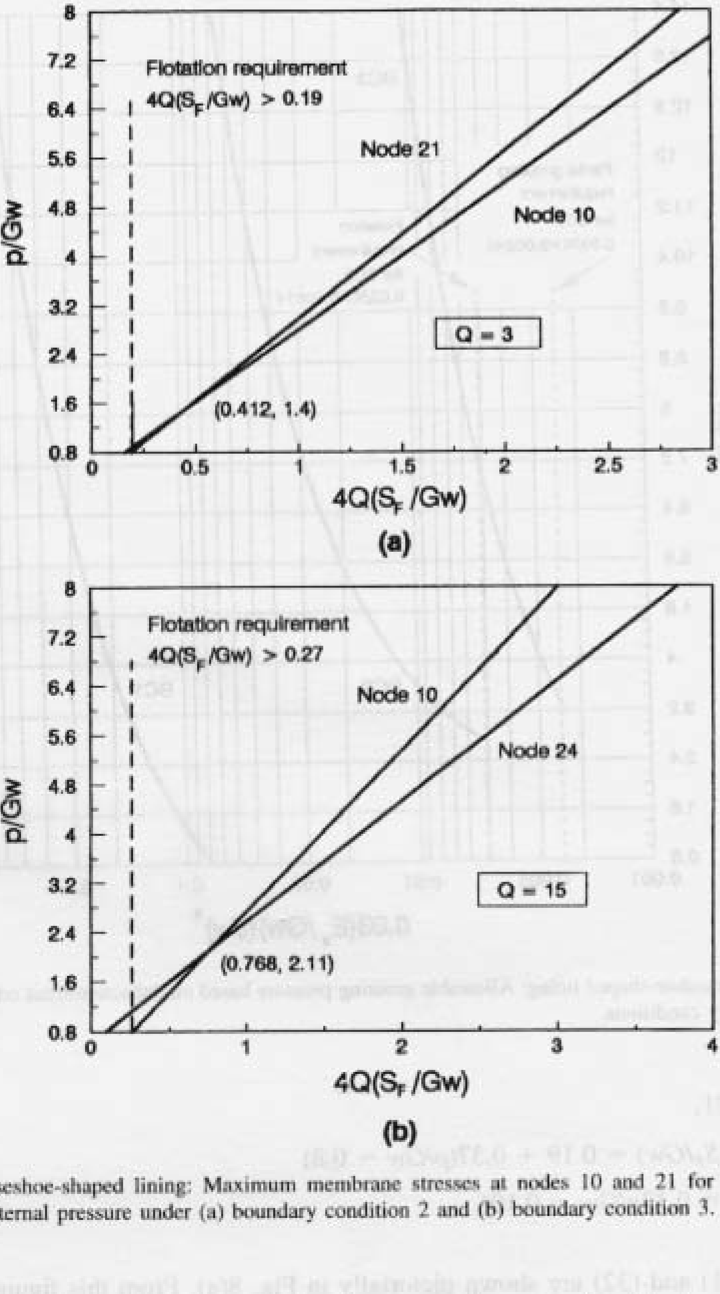


Fig. 8. Horseshoe-shaped lining: Maximum membrane stresses at nodes 10 and 21 for flotation and additional external pressure under (a) boundary condition 2 and (b) boundary condition 3.

9.3.2. Boundary condition 3: restrained at crown, invert and springing of the lining

From Table 3, it can be seen that, under boundary condition 3, the maximum membrane stress develops at node 10 for the flotation load whereas, under uniform-pressure load, the maximum stress is located at node 24. Thus, Eq. (22) must be satisfied at both nodes 10 and 24. This leads to the following two design expressions:

At node 10,

$$4Q(S_F/Gw) = 0.27 + 0.38(p/Gw - 0.8) \\ = 0.38p/Gw - 0.034 \quad (33)$$

At node 24,

$$4Q(S_F/Gw) = 0.10 + 0.51(p/Gw - 0.8) \\ = 0.51p/Gw - 0.308 \quad (34)$$

In order to visualise the effect of full-grouting load (i.e. combined flotation and uniform-pressure load), Eqs. (33) and (34) are plotted in Fig. 8(b). Evidently, Eq. (33) provides the design criterion up to a value of p/Gw equal to 2.11 (i.e. $4Q(S_F/Gw)$ equal to 0.768). After this value, it is Eq. (34) which constitutes the design expression. (As for the previous boundary case, the combined membrane stresses at all other nodes were computed, being found to be less critical than those at nodes 10 and 24.)

9.3.3. Summary of the buckling criteria

Fig. 9 summarises the results of the above two boundary conditions. It should be mentioned here that boundary condition 1 — admittedly, the most critical buckling case of all — has not been considered in the buckling analysis since a lining under this support case cannot be idealised to a two-hinged arch of equivalent radius and unrestrained length, the critical buckling stress [8] of which has been adopted as the basis for the approximate stability analysis in the present study. Moreover, it is expected that, during the installation of linings within horseshoe-shaped sewers, boundary conditions 2 or 3 are to be adopted in order to enable higher grouting pressures to be applied.

It is to be noted that the assumption of an equivalent hinged arch is conservative and can be considered as a lower-bound solution since the structural behaviour of that portion of the lining which initiates buckling (and hence is taken as the "critical" portion) is between a hinged and a fixed arch [5]. Also, the applicability of the buckling criterion as set in [8] requires an uniform pressure intensity on the arch; such a condition may very nearly be fulfilled under excess head of grout.

10. Role of additional restraints during installation

10.1. Enhancement factor

Both the maximum bending stress and the maximum deflection in a lining that arise from grouting pressure can be reduced by introducing additional restraints during installation. Similarly, additional restraints also result in an increase in resistance

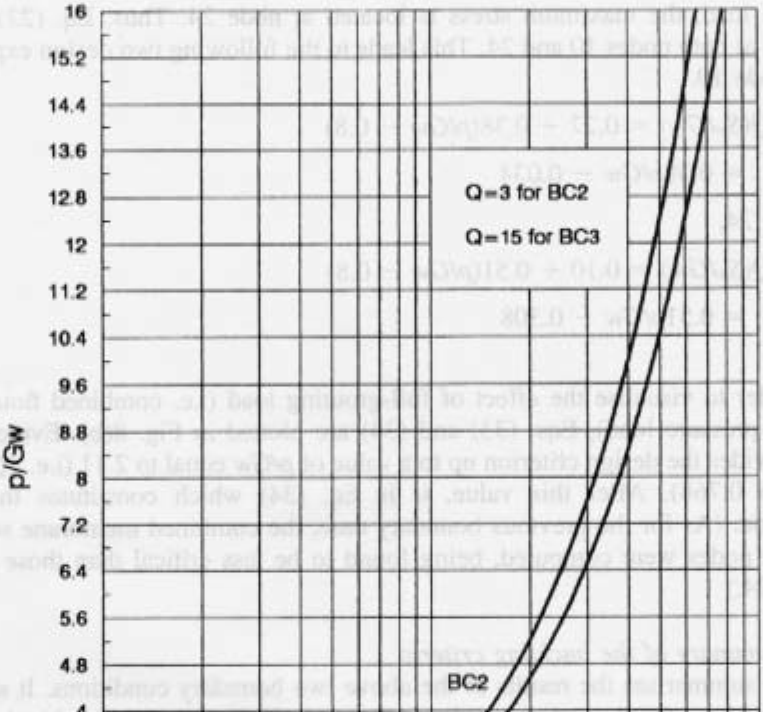


Fig. 9. Horseshoe-shaped lining: Allowable grouting pressure, based on buckling criteria, for various boundary conditions.

against buckling of the lining. This implies that an enhancement in the value of the grouting pressure can be achieved, thus ensuring adequate grouting of the annulus. This gives rise to the introduction of what can be termed an enhancement factor (EF). Here, for stress-limit and deflection-limit criteria, the enhancement factor is defined as the ratio of the allowable grouting pressure which could be applied on any particular lining using boundary case 2 or 3 to the one corresponding to boundary case 1, i.e.

$$EF_i = p_i/p_1 \qquad (35)$$

Here i corresponds to boundary cases 2 or 3. Since, in the present study, only boundary cases 2 and 3 have been considered in the buckling analysis, the corresponding enhancement factor for this third criterion is given by p_3/p_2.

Values of EF are determined for each of the stress limit, deflection-limit and buckling criteria. It has been observed that the EFs for deflection-limit criteria are much higher than their stress-limit and buckling counterparts, and, thus, will not govern the design.

10.1.1. Stress-limit criteria

The expression used to calculate the enhancement factor for stress-limit criteria can be derived from Eqs. (23)–(27) and they are given below:

$$p_1 = \frac{R - 0.3856}{0.0783} Gw \tag{36a}$$

for $0.4482 \le R \le 0.5820$,

$$= \frac{R - 0.0908}{0.1961} Gw \tag{36b}$$

for $R \ge 0.5820$,

$$p_2 = \frac{R - 0.1324}{0.0071} Gw \tag{37a}$$

for $0.1381 \le R \le 0.1563$

$$= \frac{R + 0.1885}{0.1027} Gw \tag{37b}$$

for $R \ge 0.1563$, and

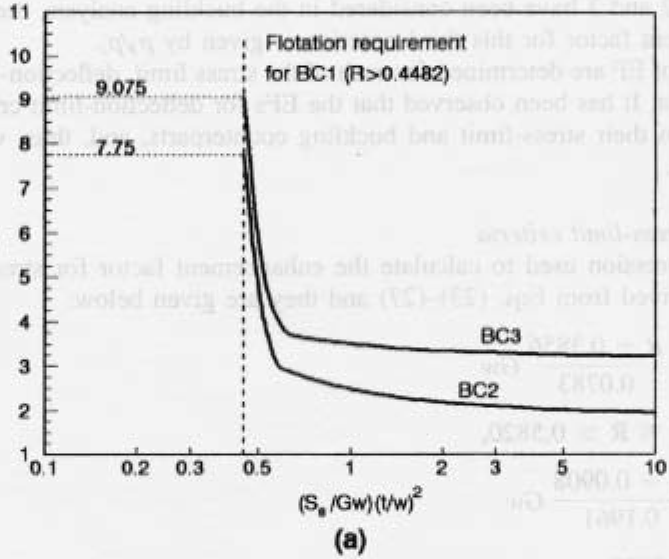
$$p_3 = \frac{R - 0.0046}{0.0611} Gw \tag{38}$$

for $R \ge 0.8$

From the above equations, enhancement factors can be calculated for boundary conditions 2 and 3; these are plotted in Fig. 10(a). From the figure, it can be deduced that the highest possible enhancement factors that can be achieved for boundary conditions 2 and 3 are 7.75 and 9.075, respectively. As the value of R increases gradually from 0.4482 to 0.5820, the value of enhancement factor sharply decreases from 7.75 to 2.99 for boundary case 2 and from 9.075 to 3.770 for boundary case 3. Beyond this range of *R*, the EF gradually decreases, attaining virtually constant values for both boundary conditions.

10.1.2. Buckling criteria

Using Eqs. (31)–(34), the enhancement factor for the buckling criterion due to the adoption of boundary condition 3 instead of boundary condition 2 during installation, is found to be as follows:



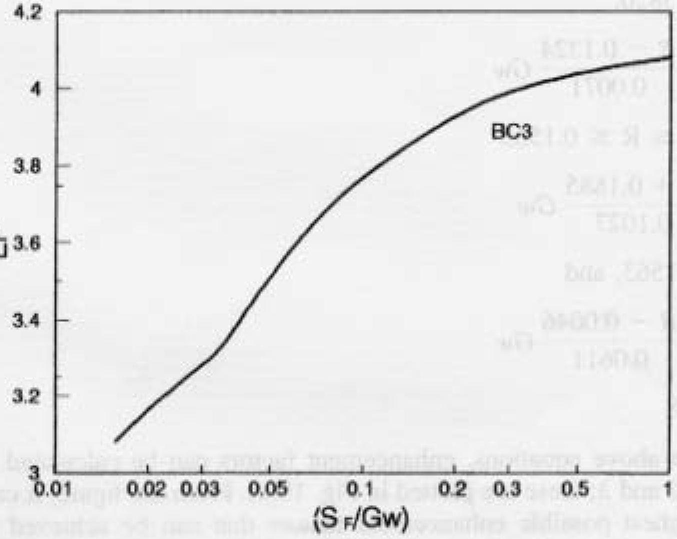


Fig. 10. Horseshoe-shaped lining: Enhancement factors for allowable grouting pressure based on (a) stress-limit criteria and (b) buckling criteria.

$$EF_3 = p_3/p_2 = \frac{0.725(60S_F/Gw + 0.308)}{12S_F/Gw + 0.106} \quad (39a)$$

for $0.0158 \leq S_F/Gw \leq 0.0343$

$$= \frac{0.824(60S_r/Gw + 0.308)}{12S_r/Gw + 0.176} \quad (39b)$$

for $S_r/Gw \geq 0.0343$

The enhancement factors corresponding to the buckling criterion are plotted in Fig. 10(b).

10.2. Reduction factors

Once a value of allowable grouting pressure is determined for any particular lining using a certain restraint set-up, a considerable reduction in the allowable thickness of the lining can usually be achieved if additional restraints are used instead. This gives rise to the introduction of another factor, called the reduction factor (RF), which is defined below.

10.2.1. Stress-limit criteria

For this criteria, the reduction factor is defined as the ratio of the lining thickness resulting from the use of boundary case 2 or 3 to the one corresponding to boundary case 1. The equation used to calculate the values of RF is as follows:

$$RF_i = t_i/t_1 \quad (40a)$$

where

$$t_i = [C_i + (p/Gw - 0.8)E_i]^{1/2}[Gw^3/S_s]^{1/2} \quad (40b)$$

and

$$t_1 = [C_1 + (p/Gw - 0.8)E_1]^{1/2}[Gw^3/S_s]^{1/2} \quad (40c)$$

with i corresponding to boundary cases 2 or 3, and other variables being defined by Eqs. (23)–(27) and Table 1.

The final equations that have been used to calculate the RFs shown in Fig. 11(a) are as follows (unlike egg-shaped or inverted egg-shaped linings, horseshoe-shaped linings always show a reduction in thickness (i.e. RF $<$ 1) as additional restraints are introduced):

$$RF_2 = \left(\left|\frac{0.1324 + 0.0071p/Gw}{0.3856 + 0.0783p/Gw}\right|\right)^{1/2} \quad (41a)$$

for $0.8 \leq p/Gw \leq 2.5$,

$$= \left(\left|\frac{0.1324 + 0.0071p/Gw}{0.0908 + 0.1961p/Gw}\right|\right)^{1/2} \quad (41b)$$

for $2.5 \leq p/Gw \leq 3.357$,

$$= \left(\left|\frac{0.1885 - 0.1027p/Gw}{0.0908 + 0.1961p/Gw}\right|\right)^{1/2} \quad (41c)$$

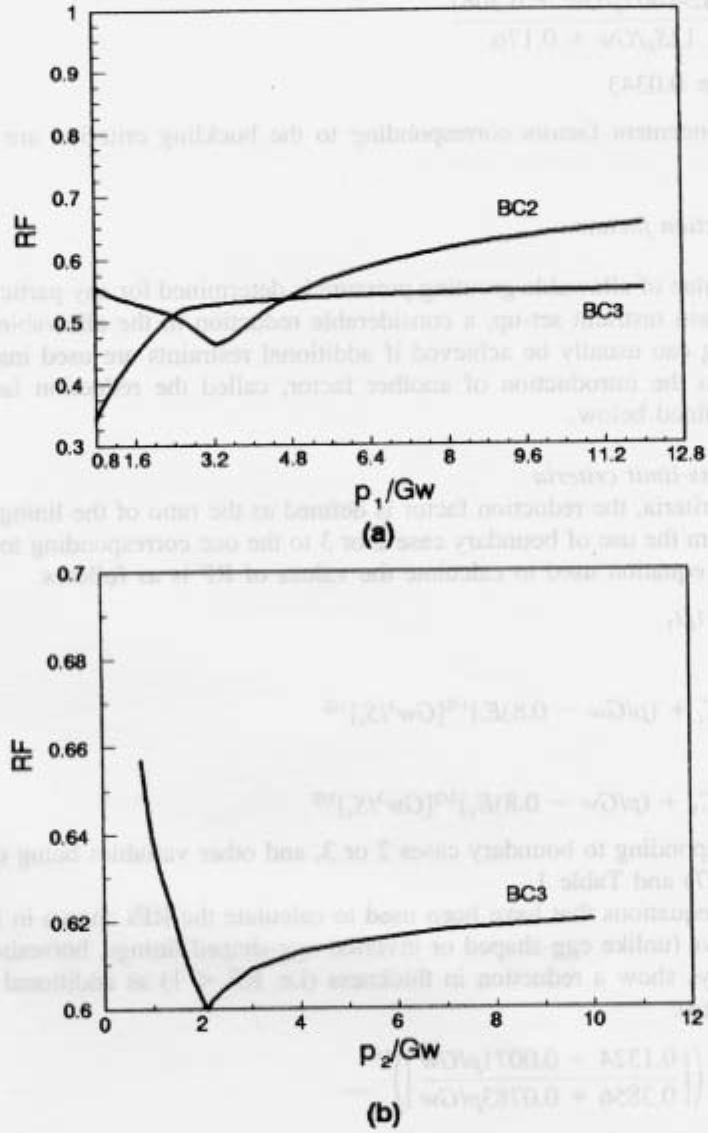


Fig. 11. Horseshoe-shaped lining: Reduction factors for minimum permissible lining thickness based on (a) stress-limit criteria and (b) buckling criteria.

for $p/Gw \geq 3.357$, and,

$$RF_3 = \left(\left|\frac{0.0046 + 0.0611p/Gw}{0.3856 + 0.0783p/Gw}\right|\right)^{1/2} \tag{42a}$$

for $0.8 \le p/Gw \le 2.5$,

$$= \left(\left| \frac{0.0046 + 0.0611p/Gw}{0.0908 + 0.1961p/Gw} \right| \right)^{1/2} \tag{42b}$$

for $p/Gw \ge 2.5$

Fig. 11(a) shows that except, for a small range of p_i/Gw, boundary condition 3 always provides reduction factors less than that of boundary condition 2. With the increase of p_i/Gw, the reduction factors attain virtually constant values for both the boundary cases.

10.2.2. Buckling criteria

In the present study, the reduction factors corresponding to buckling criteria are given by the ratio t_3/t_2 since, as for the enhancement factors considered earlier, only boundary cases 2 and 3 are considered. Using Eqs. (31)–(34), the RF is found as follows:

$$RF_3 = t_3/t_2 = \left(\left| \frac{0.38p/Gw - 0.034}{5(0.37p/Gw - 0.106)} \right| \right)^{1/3} \tag{43a}$$

for $0.8 \le p/Gw \le 1.4$

$$= \left(\left| \frac{0.38p/Gw - 0.034}{5(0.42p/Gw - 0.176)} \right| \right)^{1/3} \tag{43b}$$

for $1.4 \le p/Gw \le 2.11$

$$= \left(\left| \frac{0.51p/Gw - 0.308}{5(0.42p/Gw - 0.176)} \right| \right)^{1/3} \tag{43c}$$

for $p/Gw \ge 2.11$

The reduction factors based on the buckling criterion are shown in Fig. 11(b). As for the case of the corresponding enhancement factors, the RFs for boundary cases 2 and 3 were also determined on the basis of deflection-limit criteria. Again, stress and buckling limitations proved to be more critical for the determination of RFs, and, hence reduction factors under deflection-limit criteria are not reported here.

11. Conclusions

The proposed design curves can be used to determine the allowable grouting pressure during the installation of horseshoe-shaped sewer linings under various restraint set-ups and loading conditions. Alternatively, for a given boundary condition and known grouting pressure, the necessary lining thickness can be determined for any lining material using the currently proposed design curves and equations.

It has been shown that, by introducing additional temporary restraints before grouting around the horseshoe-shaped sewer lining, usually considerably higher grouting pressures, leading to a more reliable grouting operation, can be attained. In the study, it has been assumed that all restraints are fully effective, so that the restrained points of the lining are prevented from moving in any direction. Such ideal conditions will very nearly be realised if internal supports coupled with external packing are effectively provided.

In the case of the approximate buckling analysis, introduction of additional restraints reduced the effective length of the arch between the restraints, thus leading to a stiffer structure with higher critical buckling pressure. The presently considered buckling criterion assumes an uniform pressure intensity on the arch; the problem solved in the appendix clearly shows that such a criterion may easily be achieved if linings of adequate thickness and acceptable physical properties are employed in design; this is equivalent to the introduction of a minimum p/Gw value of, say, 3–4 (i.e. horizontal cut-offs at these values of the ordinate in Fig. 9), which implies that the head of grout is several times the size of the lining cross-section and thus means that the membrane stress state is nearly uniform.

Appendix

Design example for a typical horseshoe-shaped lining

The following design example demonstrates the use of the structural design method outlined in the present paper. The use of a particular lining material is merely illustrative.

An existing sewer is horseshoe-shaped. Its different parameters are as follows:

1. Geometrical parameters
   Overall height of the sewer = 1270 mm
   Overall width of the sewer = 1570 mm
   The minimum annulus grouting thickness to be provided = 23 mm, and the lining thickness = 12 mm

2. Material properties
   The value of the short-term Young's modulus (E_s) of the GRP lining material is equal to 20×10^6 kN/m^2 The value of allowable short-term bending stress (S_s) of the GRP lining equals 60.0×10^3 kN/m^2
   The specific weight (G) of the grout mix equals 16.0 kN/m^3

Using both boundary cases 2 and 3 as the possible temporary support systems, the allowable grouting pressure is to be determined.

Solution

From the values of the geometrical parameters, the internal dimensions of the lining (h and w) are calculated as:

$$h = 1270 - (23 \times 2 + 12 \times 2) = 1200 \text{ mm}$$
$$w = 1570 - (23 \times 2 + 12 \times 2) = 1500 \text{ mm}$$

Using the values of the material properties, the non-dimensional strength of the lining R, permissible deflection *0.03/K* and non-dimensional stiffness of the lining S_F/Gw are calculated, enabling checks on the three criteria to proceed; these are as follows for boundary case 2:

1. (Bending) stress-limit criteria: Using the values of material properties, the non-dimensional strength R of the lining is calculated.

$$R = \left(\frac{S_s}{Gw}\right)\left(\frac{t}{w}\right)^2 = \frac{60 \times 10^3}{16.0 \times 1.5}\left(\frac{0.012}{1.5}\right)^2 = 0.16$$

Using the value of R equal to 0.16 and Fig. 6 (or Eq. (26)), $p/Gw = 3.4$
2. Deflection-limit criteria:

$$\frac{0.03}{K} = 0.03\left(\frac{E_s}{Gw}\right)\left(\frac{t}{w}\right)^3 = 0.03\left(\frac{20 \times 10^6}{16.0 \times 1.5}\right)\left(\frac{0.012}{1.5}\right)^3 = 0.0128$$

Using the value of *0.03/K* equal to 0.0128 and Fig. 7 (or Eq. (29)), $p/Gw = 4.14$
3. Buckling criteria:

$$S_F = \frac{1}{12}\frac{E_s}{1-\nu^2}\left(\frac{t}{w}\right)^3 = \frac{1}{12}\left(\frac{20 \times 10^6}{1 - 0.23^2}\right)\left(\frac{12}{1500}\right)^3 = 0.901$$

From this, $\frac{S_F}{Gw}4Q = \frac{0.901}{16 \times 1.5} \times 4 \times 3 = 0.45$

Using this value and Fig. 9 (or Eq. (31)), $p/Gw = 1.49$

Hence, the minimum of these three values, i.e. $p/Gw = 1.49$ (corresponding to the buckling criterion), is to be adopted as the basis for the design.

Here, $p/Gw = 1.49$, or $p = 1.49 \times 16.0 \times 1.5 = 35.77$ kN/m^2 which is equal to 2.24 m head of grout from the invert or 1.04 m head of grout from the crown of the lining.

When the above exercise is repeated for boundary case 3, the values of p/Gw under stress-limit criteria, deflection-limit criteria and buckling criteria become 2.54, 17.64 and 5.02, respectively. Thus, under boundary condition 3, the value of p/Gw equal to 2.54 (corresponding to the (bending) stress-limit criterion) governs with allowable grouting pressure equal to 60.96 kN/m^2.

References

[1] Arnaout S, Pavlović MN. The structural behaviour of egg-shaped sewer linings with special emphasis on their current problems and research. In: Topping BHV, editor. Non-Conventional Structures, Vol. 2 (Proceedings of the International Conference on the Design and Construction of Non-Conventional Structures) (held in London, 8–10 December 1987). Edinburgh: Civil-Comp Press, 1987:231–238

[2] Arnaout S, Pavlović MN. Studies on the structural behaviour of egg-shaped sewer linings under installation and operational conditions. Structural Engineering Review 1988;1:25–33.

[3] Arnaout S, Pavlović MN, Dougill JW. Structural behaviour of closely packed egg-shaped sewer linings during installation and under various restraint conditions. Proc. ICE (Part 2) 1988;85:49–65.

[4] Pavlović MN, Arnaout S, Hitchins D. Finite element modelling of sewer linings. In: Topping BHV, editor. Developments in Structural Engineering Computing, Vol. D. (Proceedings of the Fifth International Conference on Civil and Structural Engineering Computing, CIVIL-COMP 93) (held in Edinburgh, 17–19 August 1993). Edinburgh: Civil-Comp Press. 1993:1–11. (Reprinted (with minor changes) in Computers and Structures 1997;63:837–48.)

[5] Pavlović MN, Arnaout S, Seraj SM. Some aspects of composite circular sewer linings under installation conditions. Engineering Structures, in press.

[6] Seraj SM, Roy UK, Pavlović MN. Structural behaviour of closely packed inverted egg-shaped sewer linings during installation and under various restraint conditions. Thin-Walled Structures 1997;28:89–115.

[7] Water Research Centre. Sewerage Rehabilitation Manual. Swindon, 1983.

[8] Timoshenko SP, Gere JM. Theory of Elastic Stability. New York: McGraw-Hill, 1961.