Application of the compressive-force path concept in the limit state design of footings and retaining structures

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SYNOPSIS

In the past, physical models have been proposed, in compliance with the Compressive-Force Path concept, for the practical design of various statically determinate or indeterminate reinforced- and prestressed concrete structures, made from all ranges of concrete strengths; the ensuing design resulted in economic and safer solutions. The work presently described aims at extending these models so as to encompass reinforced concrete footings and retaining structures, as well.

1. INTRODUCTION

The present-day design of various types of structural concrete members are mostly based on empirical design equations, and not on rational physical models, which enable engineers to develop a better understanding of the actual structural behaviour. In this regard, the unsatisfactory nature of the design provisions of the present Codes becomes apparent from their collection of complex, restrictive empirical equations. It would appear that it is due to the existence of a general lack of understanding, at the material- and the structure- level, that the design procedures for structural concrete, put forward by the current Codes, are unnecessarily complicated and do not always yield safe design solutions.

Although new concepts have recently evolved in the general field of structural concrete design, most of these deviate very little from the basis on which present-day design is founded; and, thus, they carry implied assumptions which, in many cases, are incompatible with the fundamental properties of concrete. The concept of Compressive-Force Path (CFP), on the other hand, departs radically from the established design concepts. This paper points at the conflicts between the traditional concepts forming the backbone of current design procedure and the actual structural behaviour.

The present study aims at introducing, admittedly tentative, physical models - in compliance with the CFP concept, for the ultimate limit state design of various footings and retaining structures.

2. TRADITIONAL CONCEPTS

2.1 General

The current design models are explicitly based on the truss analogy concept and thus very much dependent upon the residual strength of cracked concrete. But in the recent past, the widely held view that concrete exhibits strain-softening material characteristics under any state of stress has been challenged by Kotsovos (1983) with experimental evidence which has indicated that concrete as a material is brittle in nature. He has shown that conventional strain-softening response is not a material characteristics, as extensively considered, but merely a "descriptor" of secondary testing effects. Clearly, the above findings have significant implications for current analysis and design as it strikes at the heart of the present day approach.

2.2 Members in flexure

It is usually considered that behaviour of an element of concrete in the compressive zone of a reinforced concrete (RC) member in flexure is described by stress-strain characteristics as established from tests on concrete specimens such as cylinders or prisms under uniaxial compression. Typical stress-strain curves providing a full description of the behaviour of such specimens are given in Figure 1, which indicate that a characteristic feature of the curves is that they comprise an ascending and a gradually descending, the strain-hardening and the strain-softening, respectively, response of concrete in uniaxial compression. The most significant feature of concrete behaviour is the abrupt increase of the rate of lateral expansion that the specimen undergoes when the load exceeds a level close to, but not beyond, the peak level. This level is the "minimum volume level" marking the beginning of a dramatic volume dilation which follows the continuous reduction of the volume of the specimen occurring to this load level. The variation of the volume of the specimen under increasing uniaxial compressive stress is also shown in Figure 1. It is important to note that, although the curves shown in Figure 1 describe the deformational response of concrete in both the direction of loading and at right angles to this direction, it is only the former which is used by the "plane sections" theory for the description of longitudinal stress distribution within the

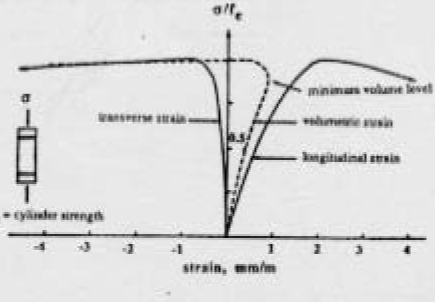


Fig. 1. Uniaxial stress-strain curves

compressive zone of the beam cross-section of an RC member.

Kotsovos (1982) carried out tests on RC members which demonstrated that in spite of the prominence given to them in flexural design, the post-ultimate uniaxial stress-strain characteristics cannot describe the behaviour of an element of concrete in the compressive zone of an RC beam in flexure. It is important to emphasize that the development of triaxial stress conditions is a key feature of structural concrete behaviour only at the late stages of the loading history of an RC member.

2.3 Members in shear

Shear capacity of an RC beam is widely defined as the maximum shear force that can be sustained by a critical cross-section with shear reinforcement being provided in order to carry that portion of the shear force that cannot be sustained by concrete alone. The amount of reinforcement required for this purpose is assessed by using one of a number of available methods invariably developed on the basis of the "truss analogy" concept (Ritter, 1899 and Mörsch, 1909), the latter stipulating that an RC beam with shear reinforcement behaves as a truss (Figure 2) once inclined cracking occurs.

A prerequisite for the application of "shear capacity of critical sections" in design appears to be (by implication) the widely accepted view that the main contributor to shear resistance is the residual strength of "cracked" concrete below the neutral axis described by the strain-softening characteristics of the material in combined compression and tension and affected by "aggregate interlock" (Fenwick and Paulay, 1968). This is because only through aggregate interlock can the cracked web be the sole contributor to the shear resistance of an RC T-beam, as specified by current code provisions. Besides, the concept of the "shear capacity of critical sections" is itself a prerequisite for the application of the "truss analogy" since it is the loss of the shear capacity below the neutral axis that the shear reinforcement is considered to counter-balance.

Kotsovos (1987a) and Kotsovos et al. (1987) demonstrated that in the absence of shear reinforcement, the main contributor to the shear resistance of an RC beam at the ultimate limit state is the compressive zone, with the region of the beam below the neutral axis making an insignificant, if any, contribution. These test results clearly render the widely held views dubious and in contrast indicate that, "truss" behaviour is not a necessary condition for the beams to attain their flexural capacity once their "shear capacity" is exceeded. The role of so-called" aggregate interlock" mechanism is also

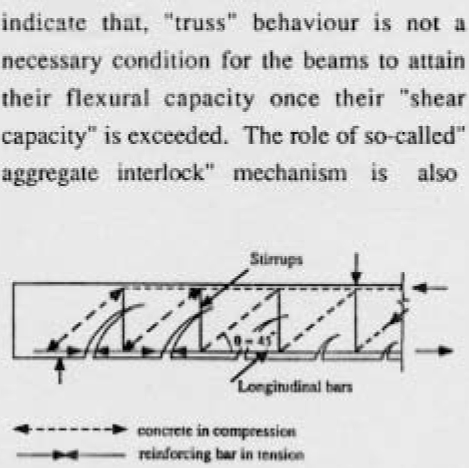


Fig. 2. Modelling by truss-analogy

questioned by Kotsovos (1987a) and Seraj et al. (1993b).

4. CFP CONCEPT

It was the consideration of the presence of multiaxial stresses in a concrete structure (Kotsovos 1987b), the recognition of concrete as a brittle material (Kotsovos 1984), and the concerns regarding the shortcomings associated with such concepts as the "truss-analogy" and the "aggregate interlock" mechanism, that led to the introduction of the CFP concept (Kotsovos 1988b).

A full description of the model together with design examples and verifications are available in other publications (Kotsovos and Lefas, 1990; Lefas et al., 1990 and Seraj et al.,1992a, 1992b, 1993a, 1993b),where it has been shown that the resulting design solutions are not only significantly more economical but also safer than those obtained by using the methods recommended by current Codes of practice.

4.1 Physical model for RC and PSC members

The load-carrying capacity of a structural concrete member, on the basis of the CFP concept, is associated with the strength of concrete in the region of the paths along which compressive forces are transmitted to the support. Figure 3 shows the "frame-like" physical model developed for the design of RC (Kotsovos and Lefas, 1990) and PSC (Seraj et al., 1993a, 1993b) beams. The path of the compressive force may be visualized as a "flow" of compressive stresses with varying section perpendicular to the path direction, the compressive force representing the resultant of the stresses at each section as shown in Figure 4. Failure has been shown to be related to the presence of tensile stresses in the region of the

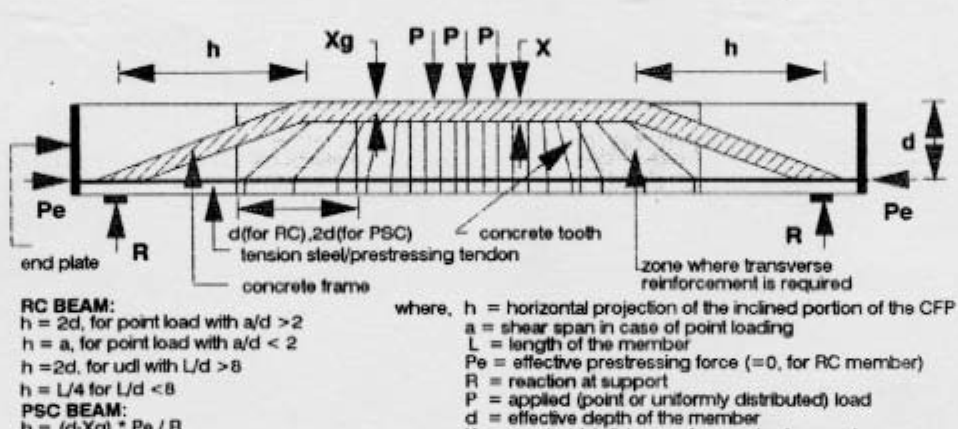


Fig. 3. CFP model for RC and PSC beams

path and such stresses may develop due to a number of causes, the main ones being associated with changes in the path direction, the varying intensity of the compressive stress field along the path, bond failure at the level of the tension reinforcement between two consecutive flexural inclined cracks, etc. It can be seen from Figure 3 that the model comprises a concrete frame with inclined legs, providing a simplified but realistic description of the shape of the CFP, and a number of "teeth" representing concrete cantilevers, which form between consecutive flexural or inclined cracks within the beam web under increasing load.

4.2 Failure criterion

In order to apply the CFP model in design, it is essential to complement it with a failure criterion capable of yielding close predictions of the load-carrying capacity of an RC member. Such a criterion may take the form of the well-established relationship between the strength of an RC beam (expressed as the moment corresponding to the failure load) and shear span or length - to - depth ratio, as depicted by the curve of Figure 5, Kani (1964). The figure indicates that the curve is essentially divided into four portions, each one corresponding to a particular type of behaviour characterized by a particular mode of failure. It may be also be interesting to note that, while the horizontal portions of the curve describe the ultimate moment of resistance of a section in pure bending, the inclined portions essentially reflect the effect of the shear force on the maximum moment that can be sustained by the section.

An analytical expression of the inclined portion of the curve for $a/d > 2$ ($L/d > 4$) has been proposed by Bobrowski and Bardhan-Roy (1969) and this, as slightly modified by

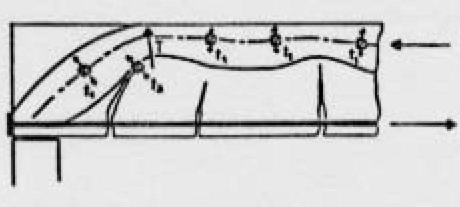


Fig.4. Schematic representation of the CFP

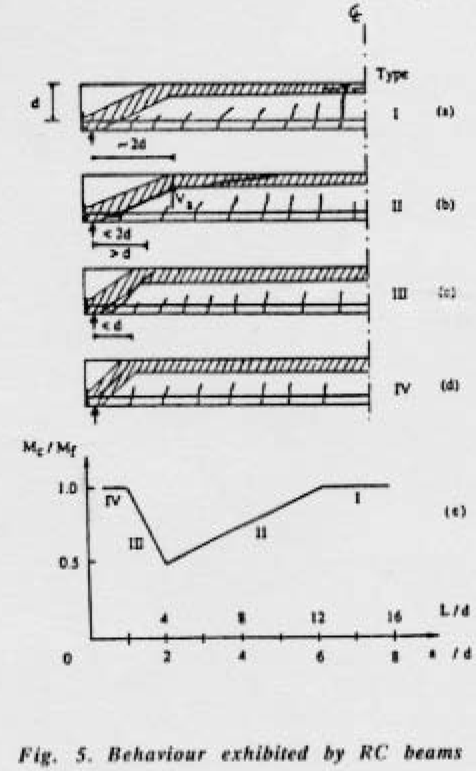


Fig. 5. Behaviour exhibited by RC beams without shear reinforcement

Kotsovos and Lefas (1990), is as follows:

$$M_c = 0.875sd\,(0.342b_l + 0.3(M_f / d^2)\,(z/s)^{1/2})\;(16.66/\rho_w f_y)^{1/4} \qquad (1)$$

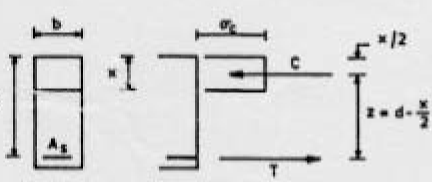
where, M_c is moment at failure load (Nmm), b_l is effective width (equal to section width for rectangular section), M_f is flexural capacity (Nmm), ρ_w is tension steel ratio, s is distance from support of cross-section at which M_c is calculated (mm) (equal to shear span for point loading, and 2d (in RC members) or h (in PSC members) for uniformly distributed loading), z is lever arm and f_y is characteristic strength of tension steel.

The use of the above failure criterion in design involves (a) designing the horizontal member of the frame so as to be capable of sustaining a compressive force C such that C = T = M/z (see Figure 6) and (b) checking whether the moment (M_a) applied at the critical cross-section s is larger or smaller than M_c. It involves the following steps:

(i). Select cross-section s; (ii) Find moment M_a at cross-section s due to applied loading; (iii) Design cross-section s to sustain a given M_f. (iv). Determine M_c from Equation 1. If $M_c > M_a$ only nominal stirrups would be needed. Otherwise, if $M_c < M_a$, either increase area of tension steel (thus increasing M_c to a level greater than or equal to M_a), or increase the cross-section; the alternative is to provide transverse reinforcement in accordance with the requirements described, in brief, below.

4.3 Assessment of transverse steel

If the conditions for failure are fulfilled before flexural failure occurs, one of the design solutions that will allow the beam to attain its flexural capacity involves the provision of transverse reinforcement. It is important to note that such reinforcement is only required, for members with shear span - to - depth ratio (a/d for point loading and M/Vd for udl) > 2, in the region of the joint of the horizontal and inclined members of the frame, with a nominal amount - as prescribed by current Codes - being sufficient in the remaining portions of the member; for members with 1 < a/d < 2, transverse reinforcement has to be extended up to the support as explained elsewhere (Kotsovos and Lefas, 1990). For a/d < 1, the member has to be designed as a deep beam (Kotsovos, 1988a), a case not encountered in foundation structures.



$M_f = T.z = C.z$

where

C	$= b \times \sigma_c$
T	$= A_s f_y$
σ_c	$= 0.8\ f_{cyl} = 0.64 - 0.67\ f_{cu}$
f_{cyl}	= cylinder concrete strength
f_{cu}	= cube concrete (characteristic) strength
f_y	= characteristic strength of tension steel

Fig. 6. Design of horizontal member of frame in the CFP model

In RC and PSC members, significant internal tensile actions may develop, for equilibrium purposes, within both the regions where the CFP changes direction and, for the case of point loading, the horizontal portion of the path in the region of point loads, to sustain tensile stresses that may develop when bond failure occurs between two consecutive flexural or inclined cracks. Since it is very unlikely that bond failure will occur in RC footing and retaining walls as they are subjected to uniformly distributed load, the method of assessing such reinforcement is not included here and may be found elsewhere (Seraj et al., 1993a, 1993b).

5. CFP METHOD IN THE DESIGN OF FOOTINGS AND RETAINING WALLS

The use of the CFP method in the design of RC foundation and retaining structures involves the development of suitable physical models representing the geotechnical structures concerned. A single footing or a retaining wall may be considered as a statically determinate structure cantilevered about the column and the wall base, respectively. On the other hand, a combined footing supporting two or more columns is essentially an indeterminate continuous beam.

5.1 Use of CFP model to cantilever structures

A cantilever beam subjected to a point loading at its free end can be designed as a simply-supported beam subjected to a point-loading at midspan, since the fixed-end conditions of the cantilever beam are similar to the conditions of the mid-span cross-section of the simply-supported beam (Figure 7). The same model can be used for the design of a structural concrete wall subjected to horizontal loading. Figure 7 shows the application of the model of Figure 3 to cantilever beams subjected to point load at its end. The same model can be used for the design of a structural concrete wall subjected to horizontal loading.

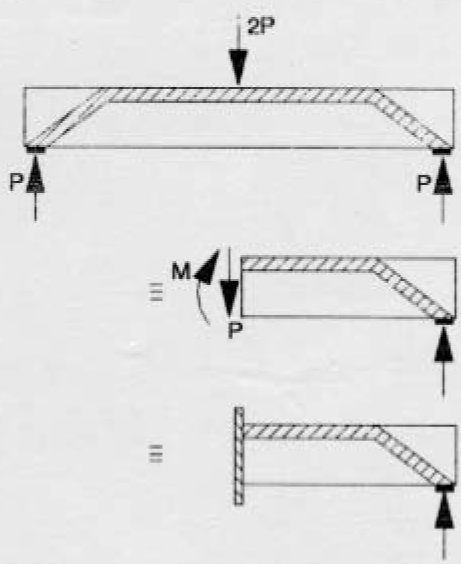


Fig. 7. CFP model for cantilever member

5.1.1 PHYSICAL MODEL FOR SINGLE COLUMN FOOTING

A single column footing subjected to uniform soil pressure may be modelled as a cantilever wall subjected to a point loading (equivalent to the sum of the net soil pressure on its cantilever portion) acting at the middle of the overhang segment. The physical model is shown in Figure 8. The part of the footing not catered for in the model may be designed similar to the horizontal member of the frame.

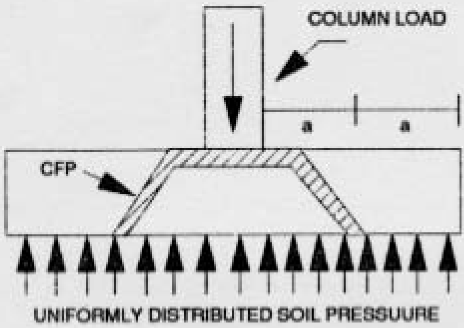


Fig. 8. CFP model for single footing

5.1.2 PHYSICAL MODEL FOR RETAINING WALLS

Retaining walls subjected to lateral earth pressure may be modelled as a cantilever beam subjected to equivalent point load at its one-third height as shown in Figure 9. In case of surcharge load at the ground level, the point of application of the load has to be re-adjusted. Usually shear is seldom a problem in a retaining wall and it exhibits type I behaviour as shown in Figure 5. As for a single footing, the portion of the retaining wall above the point load may be designed following the method narrated in Figure 6 considering a path similar to in Figure 7.

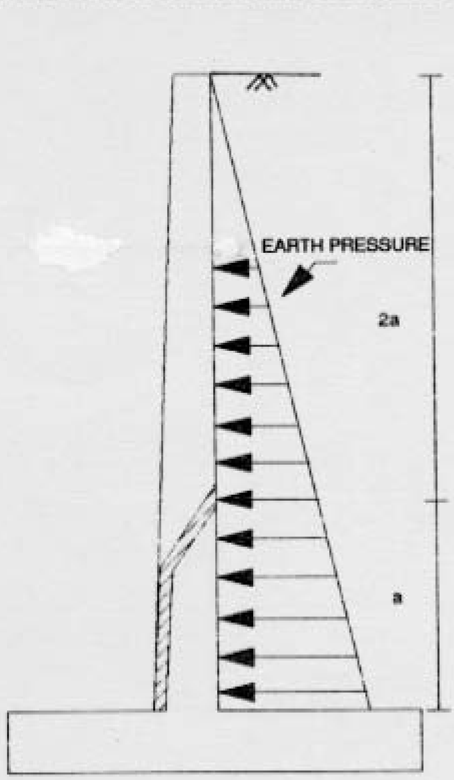


Fig. 9. CFP model for retaining wall

5.2 Use of CFP physical model to indeterminate structures

This method of design may be extended to skeletal structural concrete configurations with the frame model of Figure 3 being used to represent elements between consecutive points of inflection and the regions including such points being modelled as "internal supports". The design of the internal support, in case of skeletal structures, involves the provision of transverse reinforcement to counteract the tensile forces ("reactions") acting in that region. Considering the fact that concrete is very weak in tension, only a very minor or no contribution of concrete may be considered in designing such internal supports.

5.2.1 PHYSICAL MODEL FOR COMBINED FOOTING

In Figure 10, the above mentioned approach has been adopted in the modelling of a combined footing under the joint action of point load from three columns and uniform soil pressure from below. Initially, the combined footing has been transformed into a number of determinate structural elements. (The elastic bending moment diagram based on the design load is also shown in this figure.) In this case, however, the design should be complemented so as to allow for the interaction between different determinate portions. The latter interaction is modelled as an "internal support" where the reaction is equal to the shear force that develops in the section through the point of inflection. It is to be noted that the shape of the physical model for such determinate members, subjected either to point or uniformly distributed loading, can be ascertained by adopting the tied-frame model shown in Figure 3.

6. CONCLUSION

A reappraisal of the current design concepts, rather than changes in the design equations, should be a prerequisite for any future Code revision. By introducing new design models the CFP method may be extended to a more realistic ultimate limit state design of RC geotechnical structures .

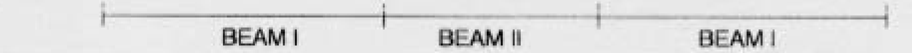


Fig. 10. CFP model for combined footing

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