STOCHASTIC APPROACH IN THE DEVELOPMENT OF RESPONSE SPECTRA BASED ON SIMULATED EARTHQUAKES

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ABSTRACT - In the present study efforts have been made to develop response spectra, which may be directly used in the dynamic analysis of a structure. Again, a suitable numerical co-efficient may be derived from those response spectra for using in equivalent static analysis. To develop response spectra a large number of earthquake ground motion records on different soil types are necessary. Bangladesh does not have any earthquake ground motion records although it falls within a seismically active zone. In case where earthquake records are not available, efforts have to be made to generate synthetic earthquakes, accomodating the properties of different soil types. To generate the synthetic earthquakes Kanai-Tajimi power spectra and Shinozuka-Sato envelope have been chosen. Response spectrum for a given earthquake record is quite irregular and has a number of peaks and valleys. Efforts have been made to construct smooth response spectra for various soil types from synthetic earthquakes. The elastic spectra of each of the synthetic accelerograms has been obtained, and then averaged to get the shape of the smoothed spectra, considering various dampings of structures. Design spectra are presented in combination of smooth curves and straight lines. Since Bangladesh lacks heavily on seismic instrumentation, it is expected that the generated response spectra based on synthetic earthquakes will form a basis for satisfactory structural analysis.

NOTATION

ζ_g	Damping ratio
w_g	Predominant frequency
G_0	Spectral density at zero frequency
ϕ	Random phase angle distributed uniformly between 0 and 2π
$x(t)$	Stationary waveform
$\psi(t)$	Deterministic envelope function
$y(t)$	Nonstationary waveform
$G(t,w)$	Two-sided instantaneous power spectrum
$n(t)$	Random process with zero mean and variance of unity
α, β	Parameters for envelope function
ϕ_1, ϕ_2, ϕ_3	A set of random numbers
S	Spectral ordinate
a,b	Constants
T	Time period of structure
T_{sm}	Time period corresponding to maximum response

INTRODUCTION

Recently, structural engineers have been giving more and more attention to the design of buildings for earthquake resistance. A great deal of time and effort has gone into the development of better methods of design. This involves a sound knowledge of earthquakes in conjunction with a better understanding of the forces that they exert on buildings. New concepts have been developed concerning the earthquake resistance of buildings as determined by their ability to absorb the energy input from the earth vibration. Earthquake-resistant design is an evolutionary one and, although great progress has been made since seismic design was made mandatory by various building codes, it is still not completely understood. Among various unknown or less known factors, the conversion of dynamic forces to static forces is, perhaps, one of the areas where additional works may be conducted. Notwithstanding the necessity of looking into the research area of arriving at equivalent static loads from their dynamic counterparts, it is believed that satisfactory design based on equivalent static forces can only be undertaken once all the factors, equations and curves that constitute such methods are realistically derived.

No ground motion record is available in Bangladesh and a few exist in the subcontinent (Chandrasekaran et al., 1997). Thus, it is difficult to obtain a generalised shape of the average spectra. Keeping this in mind, a set of synthetic accelerograms has, therefore, been simulated for normalised peak ground acceleration of different soil conditions. The elastic spectra of each of the synthetic accelerograms have been obtained, and then averaged to get the shape of the smoothed spectra, considering various damping of structures. The damping ratios of 2 percent, 5 percent and 10 percent of critical damping, considered in the present study, roughly correspond to welded steel/presstressed concrete/reinforced concrete (highly cracked), bolted and/or riveted steel/reinforced concrete (highly cracked) and reinforced masonry structures, respectively (Srivastava and Basu, 1982). In this study efforts have been made to develop a computer program to generate simulated earthquakes. To generate the synthetic earthquake, Kanai-Tajimi (K-T) (Molas and Yamazaki, 1995) power spectra which can take the effect of soil type in the nonstationary time series (Shinozuka and Sato, 1967) envelope has been chosen. Efforts are also made to construct the smooth response spectra for various soil types from synthetic earthquakes.

METHODOLOGY

To generate synthetic earthquakes, a computer program has been developed in FORTRAN which accommodates K-T power spectra and Shinozuka-Sato envelope. The detailed theoretical background needed to develop the program is mentioned in subsequent sections. The flow chart of the computer program is given in Figure 1. The input parameters that have been used to generate synthetic earthquakes for the present study are listed in Table 1. Figure 2 shows some of the typical generated earthquakes for each of the four soil types. In total 160 earthquakes were generated by varying the parameters as stated in Table 1. Generation of such a huge amount of earthquakes was deemed essential for satisfactorily construction of response spectra via simulation. Whereas real earthquake records contain both P (Compression-Expansion) wave and S (Shear) wave phases, in these figures only shear waves have been simulated. Although P-waves are always present in real earthquake records due to the inherent fault-fracture mechanism of real seismic forces, the overall impact of such forces are usually minimal in structural dynamics. Again, in contrast to real earthquake records where site effect (that is, variation of soil type) cannot be readily isolated in time domain, in the present case of simulated earthquakes, soil effect could be explicitly identified by using suitable values of natural circular frequency (w_g); thus site effects could be reflected in the simulated time history.

Table 1. Input Parameters Used to Generate Synthetic Earthquakes

Parameters	Lower Limits	Upper Limits
G_0 w_g ζ_g	50.0 cm/sec^2 5.0 rad/sec 0.15	500 cm/sec^2 35.0 rad/sec 0.55
Total Time	20.48 sec.	

To construct simulated response spectra the amplitudes of the record and the soil type have been varied. Other variables, such as magnitude and duration of earthquakes, could not be considered for lack of sufficient earthquake source and fault mechanism information in Bangladesh. To construct the simulated spectral acceleration of individual earthquake data, a computer program has been used. The elastic spectra of each of the accelerograms have been obtained, and averaged to get the smooth shape of the simulated spectra, considering the damping of various types of structures. Detailed description of construction of smooth simulated spectra has been presented in a later section.

Artificially generated earthquake

In all the data which seismology provides, the interest of structural engineers is basically centred towards those, which permit the numerical definition of the seismic action. The choice between the different existing procedures to define the seismic action is determined essentially by the quantity and quality of the seismological data available for a given area. In the case in which reliable data on the amplitude, frequency and duration of the seismic ground motion, as well as, the mechanical characteristics of the soil layers are available, the seismic force can be defined, for example, by means of artificial accelerograms. These are the accelerograms generated by using mathematical model based on the theory of the stochastic process (Ruiz and Penzien, 1969). These are then used in seismic structural analysis. Such procedures are recommended for seismic areas with no seismological data with a complete lack of seismic ground motion records.

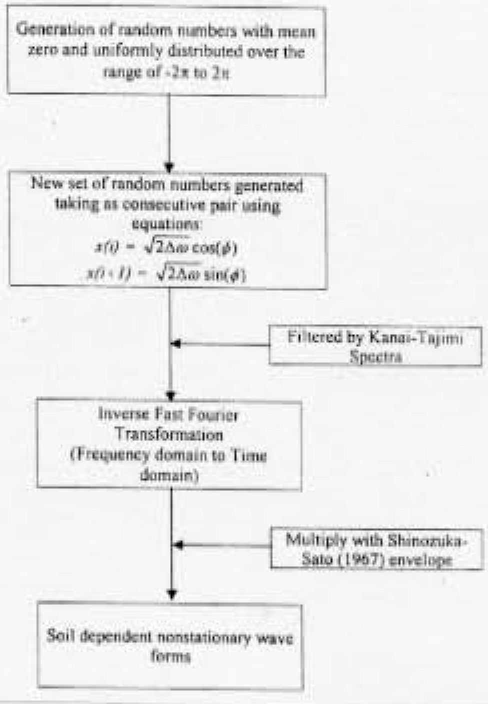

Figure 1. Schematic diagram for generating simulated earthquakes

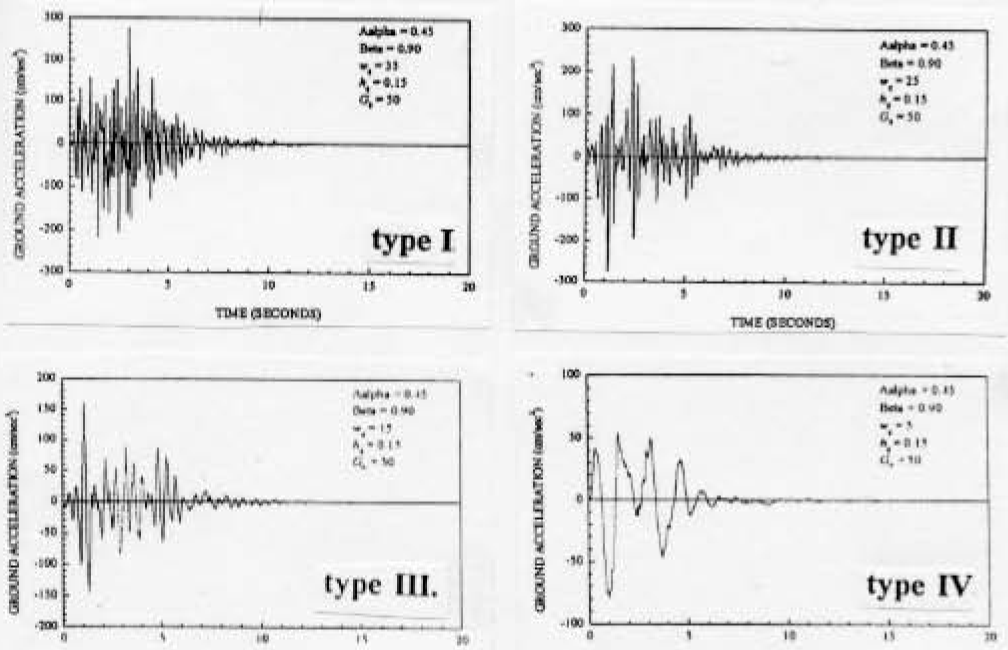

Figure 2. A selection of simulated earthquakes for four soil types

In the present study the seismic ground acceleration is modelled as a nonstationary random process.

Nonstationary model of ground acceleration

Many studies described earlier modelled strong motion part of ground acceleration as a stationary random process. However, since recorded ground motions show the nonstationarity trend very clearly, it is necessary to use nonstationary model for representing the earthquake ground motion. Generally, there are two ways to consider the nonstationarity. First, by using instantaneous power spectrum, which represents the power spectrum of ground acceleration as a function of time and frequency, nonstationary earthquake motion can be expressed as

$$x(t) = \sqrt{2\Delta w G(t,w)}\cos\phi \tag{1}$$

where $G(t,w)$ is the two-sided instantaneous power spectrum, Δw is the frequency interval and ϕ is the independent random phase angles distributed uniformly over 0 and 2π. The second way is to assume that the instantaneous power spectrum can be represented as a product of deterministic envelope function and stationary power spectrum. This assumption implies nonstationarity in intensity but stationarity or approximate stationarity in spectral characteristics, expressed mathematically as

$$x(t) = \psi(t)n(t) \tag{2}$$

$$n(t) = \sqrt{2\Delta w G(w)}\cos\phi \tag{3}$$

where $\psi(t)$ is the deterministic envelop function and $n(t)$ is a stationary random process with zero mean and unit variance.

Use of the second method has an advantage since the parameters which are controlling the shape of envelope function are independent of frequency and realisation of the nonstationary process. Therefore, in this study the second method will be employed for modelling the nonstationary ground acceleration.

Deterministic envelope funcion

From Eqns. 2 and 3, the variance of the process x(t) can be expressed as

$$Var\left[\{x(t)\}\right] = Var\left[\psi(t)\{n(t)\}\right] = \psi^2(t)Var\left[\{n(t)\}\right] = \psi^2(t) \tag{4}$$

The variance can be estimated for single record using short time-averaged (Bendat and Piersol, 1986) as

$$Var[x(t)] = \frac{1}{\theta}\int_{t-\theta/2}^{t+\theta/2} x^2(t)dt \tag{5}$$

Since the estimated variance still fluctuates, it should necessarily be fitted by a smooth function. Many function have been suggested to describe the smoothed time dependent variance. In this study, the function proposed by Shinozuka and Sato (1967) will be adopted which takes the form

$$\psi(t) = e^{-\alpha t} - e^{-\beta t} \tag{6}$$

Here for generation of simulated earthquakes, α has been varied from 0.25 to 0.45 and β has been varied between 0.50 to 0.90. For all the cases, $\alpha < \beta$.

Frequency contents of ground acceleration

Frequency contents of recorded ground acceleration are generally expressed by the power spectral density function proposed by Kanai-Tajimi (Molas and Yamazaki, 1995), and expressed here as

$$G(w) = \frac{1 + 4\xi_g^2 \left(w / w_g\right)^2}{\left[1 - \left(w / w_g\right)^2\right]^2 + \left(2\xi_g w / w_g\right)^2} G_0 \tag{7}$$

Where ζ_g, w_g and G_0 represent the damping ratio, predominant frequency and spectral density, respectively at 0 Hz. For generation of earthquake, predominant frequency has been divided into four values to represent four different soil types. The natural time period, which has been used to divide the soil into various types, are listed in the Table 2.

Table 2. Property of Kanai-Tajimi Power Spectra

Site Category	No of earthquake record generated	W_g	ζ_g	
			Maximum	Minimum
Soil Type I	40	35	0.15	0.55
Soil Type II	40	25	0.15	0.55
Soil Type III	40	15	0.15	0.55
Soil Type IV	40	5	0.15	0.55

Generation of Synthetic Accelerograms

The following procedure was adopted to develop the synthetic accelerograms. A set of random numbers, denoted by ϕ_1, ϕ_2, ϕ_3... were generated with zero mean and uniform distribution over the range of 0 to 2π. These random numbers then taken as consecutive pairs were filtered, by K-T power spectra, to correspond consecutive pair of new random numbers using Eqn. 8 and Eqn. 9 having zero mean and unit variance.

$$x(i) = \sqrt{2\Delta w G(w)} \cos(\phi) \tag{8}$$

$$x(i+1) = \sqrt{2\Delta w G(w)} \sin(\phi) \tag{9}$$

Graphical representation of this K-T power spectra filter for various soil types is shown in Figure 3. Inverse Fast Fourier Transformation (FFT) was used to convert these numbers from frequency domain to time domain. This stationary type of waveform $x(t)$ was then multiplied by Shinozuka-Sato (1967) deterministic envelope function $\psi(t)$ for conversion into nonstationary form $y(t)$. The function used for this purpose has been given in Eqn. 6 and shown in Figure 4. This nonstationary waveform $y(t)$, thus, gives the soil dependent synthetic accelerograms.

CONSTRUCTION OF SMOOTH RESPONSE SPECTRA

Response spectrum for a given earthquake record is quite irregular and has a number of peaks and valleys. The design response spectra for a particular site should not be developed from a single acceleration time history, but rather should be obtained from an ensemble of possible

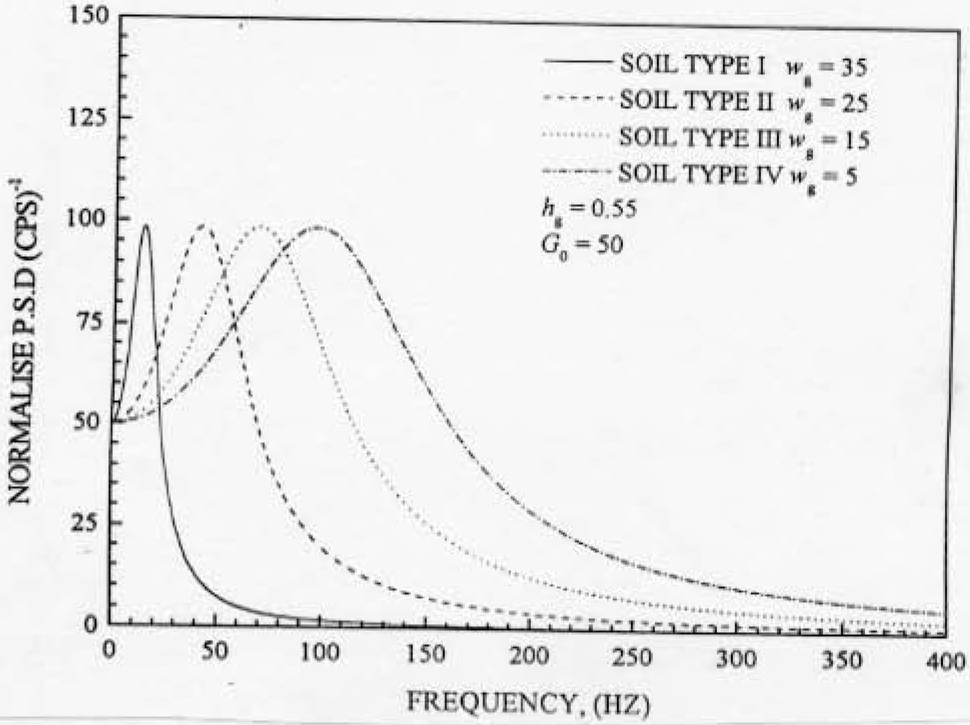

Figure 3. Normalised power spectral densities for horizontal motions for various soil types

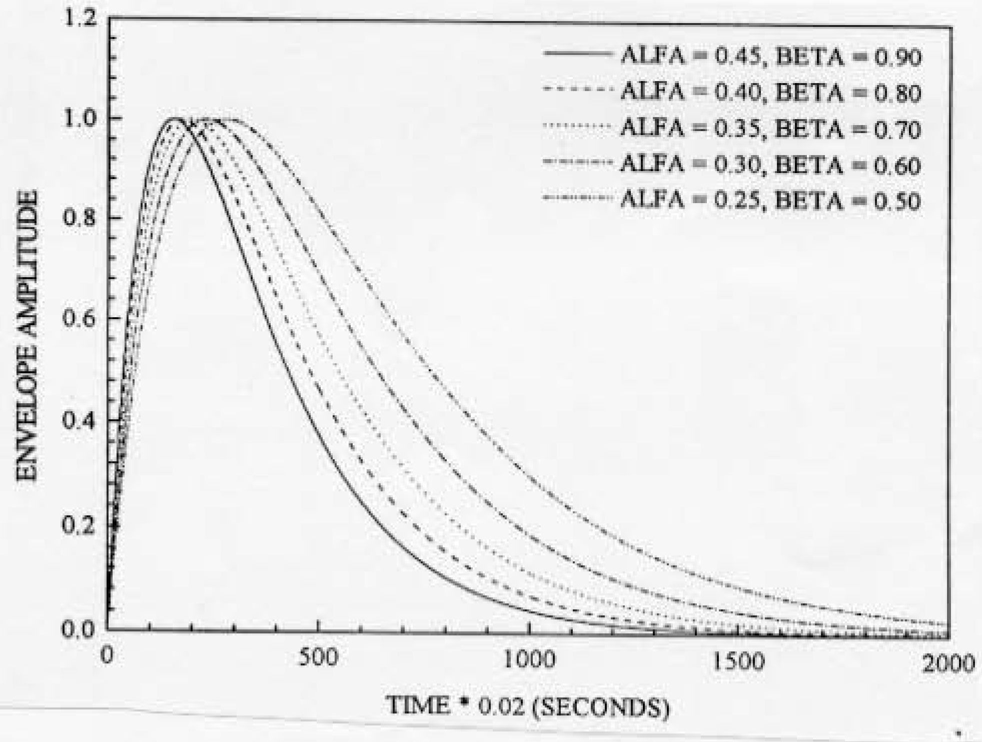

Figure 4. Shinozuka-Sato (1967) envelope for various alpha and beta

earthquake motions that could be experienced at the site. To plot the response spectra several generated earthquake records, based on the data presented in Tables 1 and 2, have been selected for the present study. Several hundred computer runs have been required to calculate the acceleration response spectra for different damping of the structure. For the present study 2 percent, 5 percent and 10 percent damping were selected. After calculating the acceleration spectra of each earthquake they were normalised against their corresponding peak value. The normalised simulated spectra were then plotted against the time period of the structure for soil type I as shown in Figure 5. A single simulated earthquake record has a particular frequency content which gives rise to the jagged, saw tooth appearance of peaks and valleys as shown in Figure 5. This feature is not suitable for design, since for a given period, the structure may fall in a valley of the response spectrum resulting in an unconservative design for an earthquake with slightly different response characteristics. Conversely, for a small change in period, the structural response might fall on a peak, resulting in a very conservative design. To come out of this problem the concept of the smoothed response spectrum has been introduced for design. The irregularities are sharp for small damping values and these smoothens as damping increases. As an example, figures for soil type I has been shown here. From Figure 5, no decision could be made regarding the location of the peak value and corresponding time period of that peak value. Plots with damping equal to 0.02 and 0.10 (Noor, 1997), however, showed that the amplitude of the spectral ordinate decreased as the damping of the structure increased. This conforms to the findings of Seed, et al. (1976).

Design spectra are presented in combination of smooth curves and straight lines. Then mean-minus-one-standard-deviation, mean, and mean-plus-one-standard-deviation have been plotted in Figure 6. This has been plotted in logarithmic scale. Statistical approach has been adopted to create a smooth spectrum in order to make it suitable for design. The mean value or median spectrum can generally be used for earthquake-resistant design of normal building structures. Use of this spectrum implies that there is a 50 percent probability that the design level will be exceeded. Structures that are generally sensitive to earthquakes or that have a high risk may be designed to higher level, such as the mean-plus-one-standard deviation, which implies that the probability of exceedence is only 15.9 percent. Structures having a very high risk are often designed for an enveloping spectrum, which envelops the spectra of the entire ensemble of possible site motions. It can be observed from Figure 6 that, the attenuation of spectral ordinate with time period plotted in logarithmic scale does not show clearly the real nature of the curve. It is clear from Figure 6 that, all sharp irregularities which was present in Figure 5 are absent in this due to the adoption of statistical procedure for the sake of clarity. Attenuation of spectral ordinate is one of the main features of the acceleration spectra plot, which has still not been identified from the Figure 6. Although at the initial stage of plotting, use of logarithmic abscissa was vital in order to identify the nature of the initial portion of the spectra, to get a clear idea of the attenuation, the spectra have been plotted in Figure 7 using linear scale between time periods of structural interest. By observing Figure 7 one can easily understand the time period at which the maximum value of spectra takes place. The mean-plus-one-standard deviation spectral shapes for the four soil types determined by the present study based on 160 synthetic earthquakes are shown in Figure 8. These spectra may be modified to use it in the seismic codal provisions. In order to achieve this modification a minimum period has been chosen, below which the spectral ordinates were kept constant to be in the conservative side. A computer program has been developed to fit the spectral curve and several computer runs were undertaken to perform this operation. These values of time period for different soil conditions have been listed in Table 3, it is apparent from the table that the natural time period of soil under considerations matched satisfactorily with the chosen time period for maximum ordinates. It is well known that resonance can occur when structural time period coincides with the natural time period of the soil beneath.

The formula $S = aT^b$ has been used to modify the simulated response spectra. S bears a constant value for $T \leq T_{sm}$. When $T > T_{sm}$, the value of S can be derived by using the formula $S = aT^b$. Here S is the spectral ordinate, a and b are constants, T is the time period of the structure, and T_{sm} is the time period corresponding to the maximum response.

Figure 9 shows the value of a and b and error in their calculation. Chi square test has been conducted during modifying the data, to ascertain the correctness of the modification. Finally, Figure 10 has been plotted to get an overall understanding of the spectral ordinates with respect to each other. Modified simulated accleration spectra for various soil types for 2 percent and ten

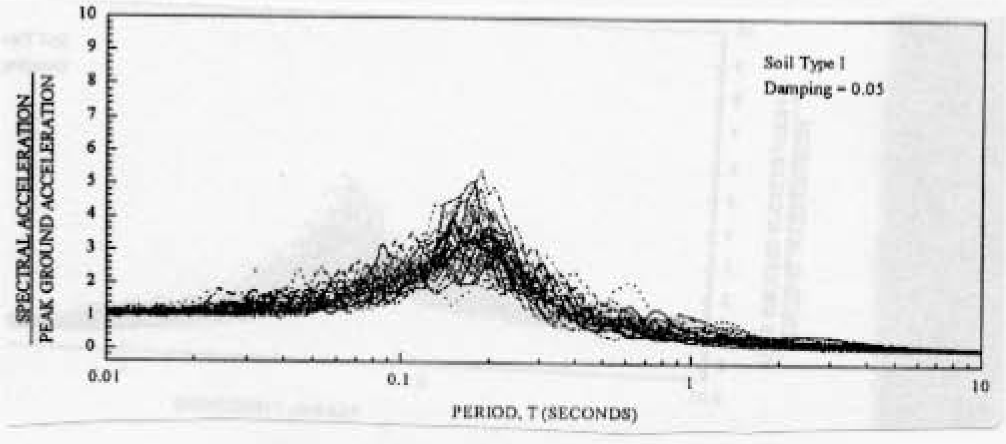

Figure 5. Normalised simulated response spectral shape of soil type I for 5 percent of critical damping

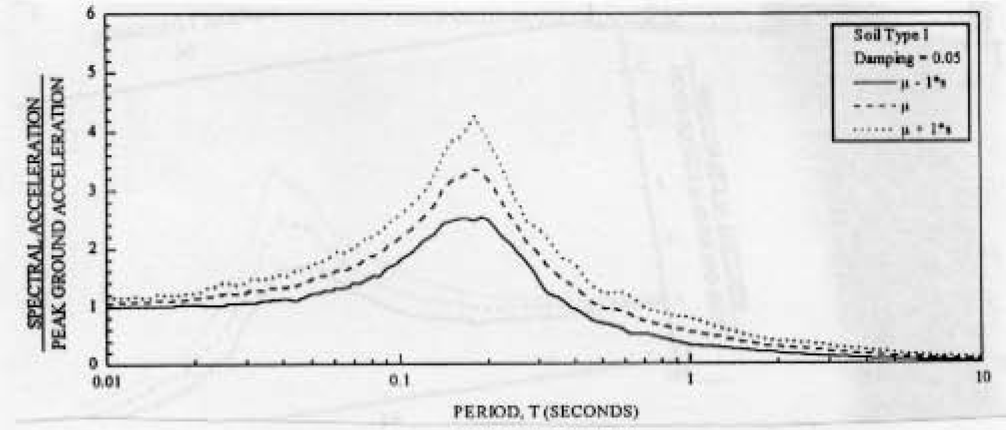

Figure 6. Simulated acceleration spectra of soil type I for 5 percent of critical damping, considering, mean-minus-one standard deviation, mean, and mean-plus-one standard deviation (in logarithmic abcissa)

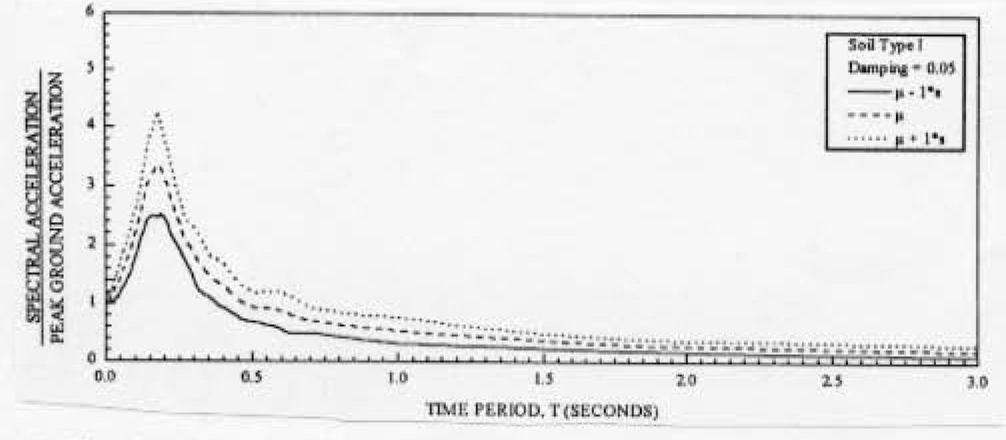

Figure 7. Simulated acceleration spectra of soil type I for 5 percent of critical damping, considering, mean-minus-one standard deviation, mean, and mean-plus-one standard deviation (in linear abcissa)

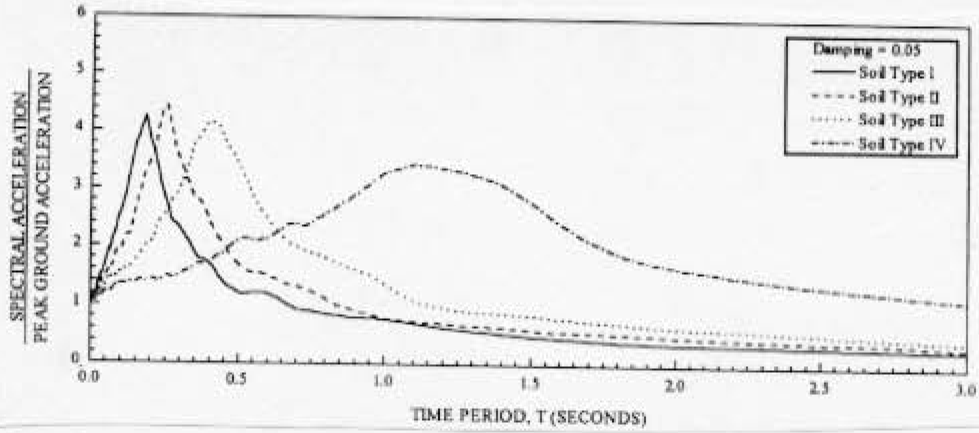

Figure 8. Simulated acceleration spectra of various soil types for 5 percent of critical damping, considering, mean-minus-one standard deviation

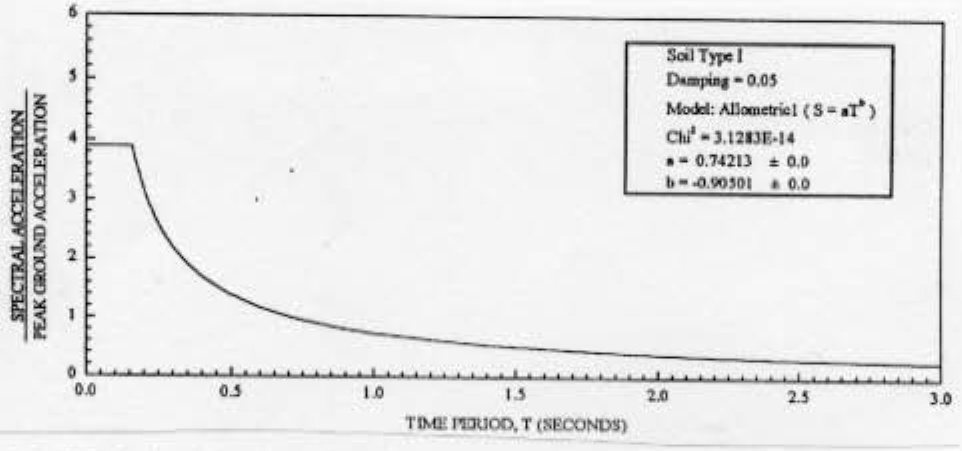

Figure 9. Modified simulated acceleration spectra of soil type I for 5 percent of critical damping

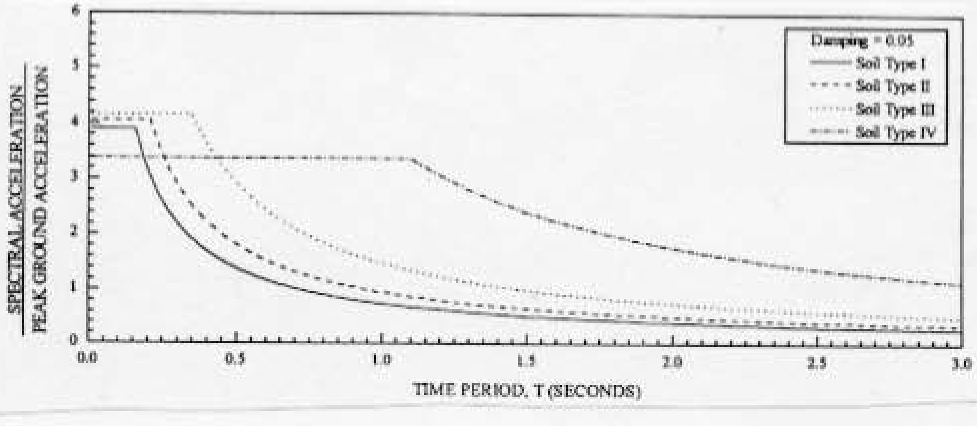

Figure 10. Modified simulated acceleration spectra of various soil types for 5 percent of critical damping

percent of critical damping is available elsewhere in Noor (1997).

Table 3 Classification of Ground Conditions for Earthquake Stations

Site Category	Time period of maximum ordinate	Definition by Natural period
Soil Type I (Rock)	0.16 sec.	$T < 0.2$ sec.
Soil Type II (Hard Soil)	0.21 sec.	$0.2 \leq T < 0.4$ sec.
Soil Type III (Medium Soil)	0.35 sec.	$0.4 \leq T < 0.6$ sec.
Soil Type IV (Soft Soil)	1.10 sec.	$T \geq 0.6$ sec.

CONCLUDING REMARKS

It is clear from the above discussion that maximum amplitude of the acceleration spectra decreased as the soil type changed from soft to rock. For larger periods, it is evident that soft soil spectral acceleration is greater than rock spectral acceleration. It is also observed that largest amplification occur near the natural time period of the soil. It has also been understood that attenuation of rock is faster than stiff soil and so is the case as the soil becomes softer. Rate of attenuation is faster for the earthquakes taken in the present study than the present-day codes

Due to the fact that Bangladesh lacks heavily on seismic instrumentation, simulated earthquakes had to be generated to arrive at site specific response spectra suitable for dynamic analysis. Whereas the spectra generated here may be used in dynamic analysis of structures, it is imperative to install suitable number of seismic stations so that in future, spectra based on site specific real earthquake records, can be developed.

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